Davenport constant and related problems

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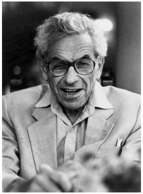
Two parts of talk

Zero-sum Problems based on the work of P. Erdős, A Ginzburg and A.Ziv :



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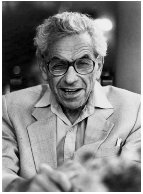

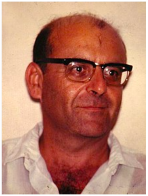

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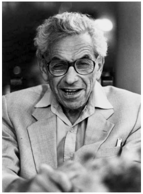
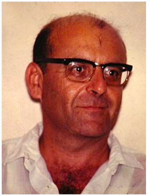


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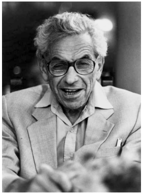
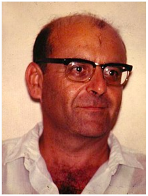


- Extremal problem related to the Davenport constant

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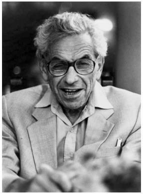
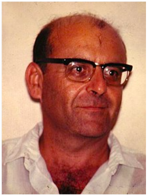


- Extremal problem related to the Davenport constant
- Zero-sum problems for random sequences

Introduction

Trivial observation

For $n \in \mathbb{N}$, and $S = a_1 \cdots a_n$ for $a_i \in \mathbb{Z}$, $\exists$ a non-trivial subsequence whose sum is divisible by n.

Length is tight: Example $S = 1^{(n-1)}$.

Erdős–Ginzburg–Ziv Theorem (1961)

Non-trivial Problem: What if there is restriction on length of the subsequence?

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Classical Result (EGZ Theorem)

Given any $n \in \mathbb{N}$, any sequence of $2n-1$ integers i.e., $a_1 \cdots a_{2n-1}$ has a subsequence of length n whose sum is divisible by n.

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This is tight: Example $S = 0^{(n-1)}1^{(n-1)}$.

Zero-sum Problem

- **Convention:** $(G, +, 0)$ is a finite abelian group.
- **Zero-sum Sequence:** A sequence (or a multiset) over G is said to be zero-sum sequence if it sums to be 0.

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  Examples:
  - $S = 1^2 3$ over $\mathbb{Z}_5$ 'YES'.
  - $S = 1.2.3$ over $\mathbb{Z}_5$ 'NO'.

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To study conditions which ensure that given sequence have non-trivial zero-sum subsequence(s) with prescribed properties.

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Example: Given n, the least $k \in \mathbb{N}$ s.t. every sequence over $\mathbb{Z}_n$ of k- length has a non-trivial zero-sum subsequence, is n itself.

Davenport Constant $D(G)$

Definition [Roger (1963)]

The Davenport Constant $D(G)$ is the smallest positive integer k such that for any sequence $x_1 \cdots x_k$ of length k over G,

$$0 \in (\{0,1\}x_1 + \cdots + \{0,1\}x_k) \setminus (\{0\}x_1 + \cdots + \{0\}x_k).$$

Example: $D(\mathbb{Z}_n) = n$.

Importance of Davenport Constant

- The Davenport constant is an important invariant of the ideal class group:
  - If O be the ring of integer over the number field and G, its ideal class group, then $D(G)$ is the max no. of prime ideals occurring in the prime ideal decomposition of an irreducible in O.

Some known results for $D(G)$

- **Olson (1969):**

$$D(\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2}) = n_1 + n_2 - 1 \text{ for } n_1 \mid n_2.$$

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- **Olson (1969):** For prime p,

$$D(\mathbb{Z}_{p^{e_1}} \times \cdots \times \mathbb{Z}_{p^{e_r}}) = 1 + \sum_{i=1}^{r} (p^{e_i} - 1).$$

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- **Conjecture [Olson (1969)]:** If $n_i \mid n_{i+1}$ then

$$D(\mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_r}) = 1 + \sum_{i=1}^{r} (n_i - 1)$$

Conjecture "NOT TRUE"

- **Geroldinger and Scneider (1992) :** For group of rank 4, Conjecture **not true** for

$$G = \mathbb{Z}_m \times \mathbb{Z}_n^2 \times \mathbb{Z}_{2n} \text{ for every odd } m, n \text{ with } m \mid n.$$

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- **Open Problem:** Whether the conjecture true for group of rank 3?

For group of rank 3

Condition A

For distinct primes p, q ($p \neq q$) and $G := \mathbb{Z}_p^3 \times Z_q$. Let $(x_1, \ldots, x_m)$, $(y_1, \ldots, y_m)$ are sequences over $\mathbb{Z}_p^3$, $\mathbb{Z}_q$ respectively with $m = p(q+2) - 2$ and

$$\begin{aligned} y_{\sum_{i=1}^{j-1} r_i + 1} = \cdots = y_{\sum_{i=1}^{j} r_i} &= j \text{ where } 1 \leq j \leq q-1, \\ \text{and } y_{r+1} = \cdots = y_m &= 0 \text{ where } r = \sum_{i=1}^{q-1} r_i. \end{aligned}$$

If $r \in [pq+1, p(q+2)-2]$ and $\sum_{i=1}^{q-1} i r_i \equiv 0 \pmod{q}$, then $S := (x_1, y_1) \ldots (x_m, y_m)$ has a nontrivial zero-sum subsequence.

Result

A. Biswas and M. (2023+)

For distinct primes p, q and $G := \mathbb{Z}_p^3 \times Z_q$. If Condition A holds true then the Olson's Conjecture holds true.

Some known results:

- **G. Bhowmik and J. Schlage-Puchta (2007):** For $p = 3$ and q being any integer not only prime.

Davenport constant for non-abelian group

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- The **Weak Davenport constant** $D(G)$ is the smallest positive integer k such that for any sequence $x_1 \cdots x_k$ of length k over G, there are $1 \leq i_1, i_2, \cdots, i_l \leq k$ s.t

$$x_{i_1} * x_{i_2} * \cdots * x_{i_l} = 1.$$

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- The **Ordered Davenport constant** $D_0(G)$ is the smallest positive integer k such that for any sequence $x_1 \cdots x_k$ of length k over G, there are $1 \leq i_1 < i_2 < \cdots < i_l \leq k$ s.t

$$x_{i_1} * x_{i_2} * \cdots * x_{i_l} = 1.$$

Note: $D(G) \leq D_0(G)$.

Known results

- **Olson and White (1977) :** For finite non-cyclic group,

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- Recall, $D_{2n} = < x, y | x^2 = y^n = (xy)^2 = 1 >$. Consider the sequence

$$S = \underbrace{y \dots y}_{n-1 \text{ times}} \ x$$

This concludes $D(D_{2n}) = n + 1 = D_0(D_{2n})$.

Known results

- Recall the dicyclic group,

$$Q_{4n} = < x, y | x^2 = y^n, y^{2n} = 1, (yx)^2 = 1 > .$$

Consider the sequence

$$S = \underbrace{y \dots y}_{2n-1 \text{ times}} \; x$$

This concludes $D(Q_{4n}) = 2n + 1 = D_0(Q_{4n})$.

Ordered Davenport constant

Let G be a finite p-group and $\mathbb{F}_p G$ is the modular group algebra. The nilpotency index of jacobson ideal of $\mathbb{F}_p G$ is called Loewy length i.e., $L(G)$. Then,

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- **Conjecture [Dimitrov (2004)] :** For any p-group G,

$$D_0(G) = L(G).$$

Known result

- **Dimitrov (2004):**Consider, the Heisenberg group of order p^3 i.e.

$$H_{p^3} =< a, b, c | a^p = b^p = c^p = [a, c] = [b, c] = 1, [a, b] = c >$$

  - Consider

$$S = (abc^{\frac{1}{2}})^{(p-1)}(ab^3c^{\frac{3}{2}})^{(p-1)}(b^{-1})^{(p-1)}(a^{-1}bc^{\frac{-1}{2}})^{(p-1)}$$

  - $D_0(H_{p^3}) = 4p - 3 = L(H_{p^3})$ for $p \equiv 3(\text{mod}4)$.

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- $G_4 =$ representation not known.

Main Result

Theorem [Godara, Joshi and M. (2023+)]

For an odd prime p, we have,

- $D_0(G_1) = p^\alpha + p^\beta + 2p^\gamma - 3 = L(G_1)$, for $\gamma = 1$.
- $D_0(G_2) = p^\alpha + p^\beta - 1 = L(G_2)$.
- $D_0(G_3) = p^\alpha + p^\beta + 2p^\sigma - 3 = L(G_3)$, for $\sigma = 1$.

Extremal problem related to weighted Davenport constant

A-weighted Davenport constant, $D_A(G)$

Let $(G, +, 0)$ be a finite abelian group.

Definition [Adhikari et al. (2006)]

For $A\ (\neq \emptyset) \subseteq \mathbb{Z}_{\exp(G)} \setminus \{0\}$, $D_A(G)$ is the smallest $k \in \mathbb{N}$ s.t. for any sequence $x_1 x_2 \cdots x_k$ with length k over G,

$$0 \in (A \cup \{0\})x_1 + \cdots + (A \cup \{0\})x_k \setminus (\{0\}x_1 + \cdots + \{0\}x_k).$$

Example: $D_{\pm}(\mathbb{Z}_n) = \lfloor \log_2 n \rfloor + 1$.

Importance of Weighted Davenport Constant

- An combinatorial interpretation of this for $G = (\mathbb{Z}_p)^n$:
  - For arbitrary $A \subseteq \mathbb{Z}_p^*$, $D_A(G)$ measures how large a sequence vector in $(\mathbb{Z}_p)^n$ can be, if the sense of 'independence' restricts the coefficients of the vectors to A.
  - **Thangadurai (2007):** If $A = \mathbb{Z}_p^*$, then

$$D_A(G) = n + 1$$

i.e. the precise dimension of it is n.

Some known results for $\mathbf{G} = \mathbb{Z}_\mathbf{n}$

- $D_{\{1\}}(\mathbb{Z}_n) = n$.
- **Adhikari, Chen, Friedlander, Konyagin, and Pappalardi (2006):**

$$\text{For } A = \{1, -1\}, D_A(\mathbb{Z}_n) = 1 + \lfloor \log_2 n \rfloor.$$

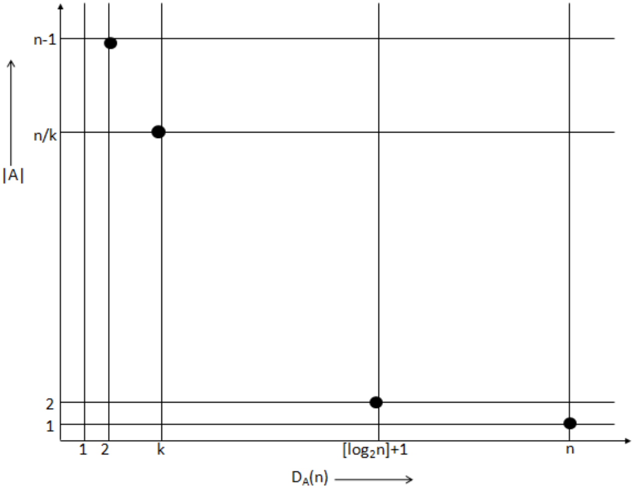
- **Adhikari, Chen, Friedlander, Konyagin, and Pappalardi (2006):**

$$\text{For } A = \mathbb{Z}_n \setminus \{0\}, D_A(\mathbb{Z}_n) = 2.$$

- **Adhikari, David, and Urroz (2008):** If $r, n \in \mathbb{N}$ and $1 \le r < n$ then

$$\text{For } A = \{1, 2, ..., r\}, D_A(\mathbb{Z}_n) = \left\lceil \frac{n}{r} \right\rceil.$$

$|A|$ verses $D_A(n)$ Graph



Extremal Problem

$f_G^{(D)}(k)$ [Balachandran and M. (2019)]

For $k \geq 2$,

$$f_G^{(D)}(k) := \begin{cases} \mathsf{min}\{|A| : A \subseteq [1, \exp(G) - 1] \text{ s.t } \; D_A(G) \leq k\} \\ \infty \text{ if there is no such } A. \end{cases}$$

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Natural Problem: Given a finite abelian group G, and $k \geq 2$,

$$\text{Determine } f_G^{(D)}(k).$$

Notation : $f^{(D)}(n, k) := f_{\mathbb{Z}_n}^{(D)}(k)$

Non-trivial Upper bounds for $f_G^{(D)}(k)$

Theorem [Balachandran and M. (2019)]

Let $G = \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_s}$, where $1 < n_1$ and $n_i \mid n_{i+1}$. For $1 \leq r < (n_s - 1)/2$ and $A = \{\pm 1, \pm 2, \cdots, \pm r\}$

- For $s = 1$, $D_A(\mathbb{Z}_{n_1}) = 1 + \lfloor \log_{r+1} n_1 \rfloor$.
- For $s > 1$,

$$\sum_{i=1}^{s} \lfloor \log_{r+1} n_i \rfloor + 1 \leq D_A(G) \leq \sum_{i=1}^{s} \lceil \log_{r+1} n_i \rceil + 1.$$

- $f_G^{(D)}(k) \leq 2(|G|^{\frac{1}{k-s-1}} - 1)$ for $s > 1$.
- $f^{(D)}(n_1, k) \leq 2(n_1^{\frac{1}{k-1}} - 1)$.

Main Results

Theorem [Balachandran and M. (2019)]

$$p^{\frac{1}{k}} - 1 \leq f(p,k) \leq 2(p^{\frac{1}{k-1}} - 1)$$

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Theorem [Balachandran and M. (2019)]

For sufficiently large prime p, we have

$$p^{\frac{1}{k}} - 1 \leq f(p, k) \leq O((p \log p)^{1/k})$$

Main Results

Conjecture [Balachandran and M. (2019)]:

For prime p,

$$f(p,k) = \Theta(p^{1/k})$$

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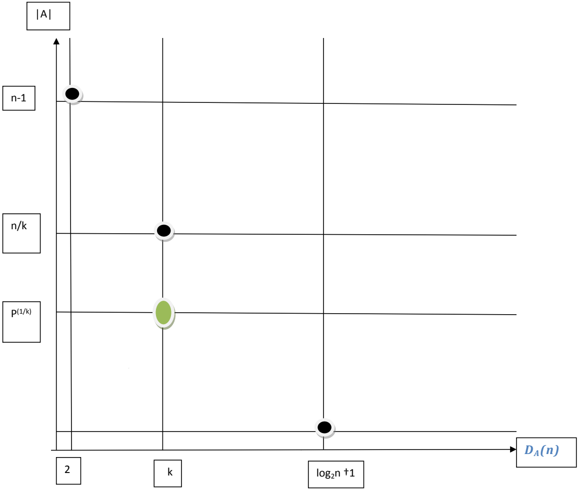
Theorem [Balachandran and M. (2023)]

For all primes sufficiently large prime p,

$$f(p,k) = \Theta_k(p^{\frac{1}{k}}).$$

$|A|$ verses $D_A(n)$ Graph



Random Sequences

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Theorem [Balachandran and M. (2021)]

Suppose $\omega(n)$ is a function that satisfies $\omega(n) \to \infty$ as $n \to \infty$.

- The following hold *whp* (as $n \to \infty$) :

$$\mathcal{X}_m \text{ is a Davenport } \mathbb{Z}_n\text{-sequence if}$$

$$m \geq \log_2 n + \omega(n),$$

$$\mathcal{X}_m \text{ is not a Davenport } \mathbb{Z}_n\text{-sequence if}$$

$$m \leq \log_2 n - \omega(n).$$

Main Results

Continue [Balachandran and M. (2021)]

- Suppose $A = \{-1, 1\}$. Then *whp* (as $n \to \infty$) the following hold:

$$\mathcal{X}_m \text{ is an } A\text{-weighted Davenport } \mathbb{Z}_n\text{-sequence if}$$

$$m \geq \log_3 n + \omega(n).$$

$$\mathcal{X}_m \text{ is not an } A\text{-weighted Davenport } \mathbb{Z}_n\text{-sequence if}$$

$$m \leq \log_3 n - \omega(n)$$

- Suppose $n = p_1 \cdots p_r$ where p_i are distinct odd primes and let $A = \mathbb{Z}_n^*$. Then if $m \geq \omega(n)$ then $\mathcal{X}_m$ is an A-weighted Davenport $\mathbb{Z}_n$-sequence *whp* (as $n \to \infty$).

Useful tools

- Graph theoretical and Combinatorials methods
- Number theoretic methods : Quadratic residue, Hardy-Littlewood conjecture
- Jenning's Theorem, Singer's Theorem
- Probabilistic methods: Janson Inequality, Markov Inequality

Future Plans

- Identify $p \neq q$ for which Condition A holds true

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- Ordered Davenport constant for complete class of p-group

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- Let $\epsilon > 0$. Suppose $\mathcal{X}_k = X_1 \ldots X_k$ is a random $\mathbb{Z}_p$-sequence. $\mathcal{A}_\epsilon := \{A : \mathbb{P}(\mathcal{X}_k \text{ an } A\text{-weighted Davenport Z-seq. }) \geq 1-\epsilon\}$. Determine
$$f^{(D)}_{\mathrm{Rand}}(p, k, \epsilon) := \min_{A \in \mathcal{A}_\epsilon} |A|$$

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THANK YOU