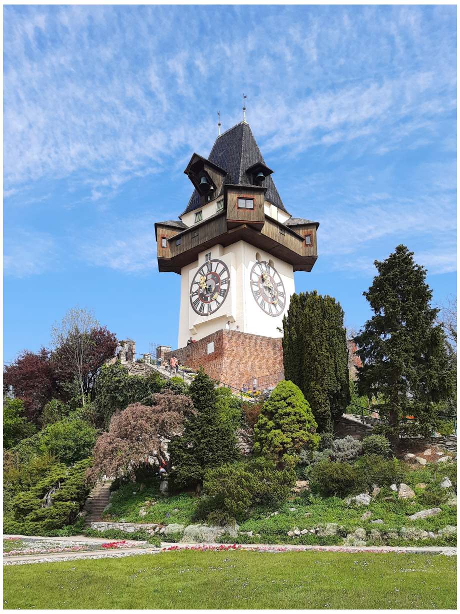
Conference on Rings and Factorizations 2023

With special sessions dedicated to **Prof. Matej Brešar** and **Prof. Sophie Frisch**
on the occasion of their 60^{th} birthdays

July 10-14, 2023
Graz, Austria



CONFERENCE BOOKLET

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CONFERENCE INFORMATION

Date

July 10-14, 2023

Place of event

University of Graz
Institute of Mathematics and Scientific Computing
Heinrichstraße 36
8010 Graz
Austria

Scientific advisory board

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Conference website

https://imsc.uni-graz.at/rings2023/

Plenary speakers

- Gyu Whan Chang (Incheon National University, Republic of Korea):
  Prime Factorization of ideals in commutative rings,
  with a focus on Krull rings
- Jim Coykendall (Clemson University, USA):
  Half-factoriality and length factoriality in monoids and domains
- Marco D'Anna (Università di Catania, Italy):
  On the Apéry algorithm for a plane singularity
- Felix Gotti (Massachusetts Institute of Technology, USA):
  Strong Types of Atomicity
- Roozbeh Hazrat (Western Sydney University, Australia):
  Leavitt path algebras
- Alessio Sammartano (Politecnico di Milano, Italy):
  On the number of relations of a numerical semigroup
- Dario Spirito (Università di Udine, Italy):
  Derived set-like constructions in commutative algebra
- Francesca Tartarone (Università degli Studi Roma Tre, Italy):
  Essential properties of Integer-valued polynomial rings
- Jurij Volčič (Drexel University, USA):
  Determinantal zeros and factorization of noncommutative
  polynomials

List of participants

Wasim Ahmed, Aligarh Muslim University, India
Amr Ali Abdulkader Al-Maktry, TU Graz, Austria
Román Álvarez Arias, Universitat Autònoma de Barcelona, Spain
Siddhi Balu Ambhore, Indian Institute of Technology Gandhinagar, India
Gerhard Angermüller, Germany
Pham Ngoc Ánh, Alfréd Rényi Institute of Mathematics, Hungary
Abu Zaid Ansari, Islamic University of Madinah, Kingdom of Saudi Arabia
Pere Ara, Universitat Autònoma de Barcelona, Spain
María Jose Arroyo, Universidad Autónoma Metropolitana, México
Sadia Arshad, National College of Business Administration and Economics, Pakistan
Paul Baginski, Fairfield University, USA
Ara Balaki
Aqsa Bashir, University of Graz, Austria
Charlie Beil, University of Graz, Austria
Nathan Blacher, University of Sheffield, United Kingdom
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Rachid Boumahdi, National Higher School of Mathematics, Algeria
Victor Bovdi, United Arab Emirates University, United Arab Emirates
Maria Bras-Amorós, Universitat Rovira i Virgili, Catalonia
Matej Brešar, University of Ljubljana, Slovenia
Federico Campanini, UCLouvain, Belgium
Jean-Luc Chabert, University of Picardie, France
Gyu Whan Chang, Incheon National University, Republic of Korea
Scott Chapman, Sam Houston State University, USA
Hyun Seung Choi, Kyungpook National University, Republic of Korea
Anna Cichocka, Warsaw University of Technology, Poland
Laura Cossu, University of Graz, Austria
Marco Giuseppe Cottone, Università di Catania, Italy
Jim Coykendall, Clemson University, USA
Radouan Daher, University Hassan II Casablanca, Morocco
Marco D'Anna, Università di Catania, Italy
Brahim El Alaoui, Mohammed V University in Rabat, Morocco

Susan El-Deken, Helwan University, Egypt
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Felix Gotti, Massachusetts Institute of Technology, USA
Nico Groenewald, Nelson Mandela University, South Africa
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Lorenzo Guerrieri, Jagiellonian University in Kraków, Poland
Gülizar Günay Mert, TU Graz, Austria
Zeeshan Haider, School Education Department, Pakistan
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Mi Hee Park, Chung-Ang University, Republic of Korea
Giulio Peruginelli, Università di Padova, Italy
Harold Polo, University of California, USA
Mara Pompili, University of Graz, Austria
Devendra Prasad, Shiv Nadar University Chennai, India
Balint Rago, University of Graz, Austria
Rameez Raja, National Institute of Technology Srinagar, India
Jutta Rath, University of Klagenfurt, Austria
Samarpita Ray, ISI Bangalore, India
Andreas Reinhart, University of Graz, Austria
Sávio Ribas, Universidade Federal de Ouro Preto, Brazil
Roswitha Rissner, University of Klagenfurt, Austria
Moshe Roitman, University of Haifa, Israel

Parackal G. Romeo, Cochin University of Science and Technology, India
Alessio Sammartano, Politecnico di Milano, Italy
Barna Schefler, Eötvös Loránd University, Hungary
Wolfgang A. Schmid, LAGA, Université Paris 8, France
Alfilgen Sebandal, Mindanao State University, Philippines / Western Sydney University, Australia
Giovanni Secreti, Università di Catania, Italy
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Steven, University of Sheffield, United Kingdom
Teerapong Suksumran, Chiang Mai University, Thailand
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Nicholas Werner, State University of New York at Old Westbury, USA
Roger Wiegand, University of Nebraska Lincoln, USA
Sylvia Wiegand, University of Nebraska Lincoln, USA
Daniel Windisch, TU Graz, Austria
Qinghai Zhong, University of Graz, Austria
Michał Ziembowski, Warsaw University of Technology, Poland

Registration desk

The registration desk at the conference venue will open on Monday, July 10, 2023 at 08:00 a.m.

Library

Location: Institute of Mathematics and Scientific Computing
Heinrichstraße 36
3rd floor
Opening hours: Monday - Friday: 9:00 a.m. - 1:00 p.m.

Internet access

Internet services are available for free. To access the internet, turn on the wireless LAN on your computer in the area of the conference premises and select the network called

"UNIGRAZguest"

This network is password-protected, so you require the following guest user name and password to access it.

Guest user name: rings2023 Password: nsaiMgrZ

Social events

Monday: Music performance

After the talks on Monday, July 10, 2023 there will be a musical performance by Maria Bras-Amorós and Daniel Windisch.

Tuesday: Reception by the Governor

On Tuesday, July 11, 2023 at 7:00 p.m., the Governor of the Federal State of Styria, LH Mag. Christopher Drexler, will host a reception for the participants of the conference in the white hall of Graz Burg (Hofgasse 15, 8010 Graz). We (the organizers) will walk together from the conference venue to the reception and you are welcome to join us. *The security team will ask for your invitation card.*

Wednesday: Guided Tour of Graz and Dinner

On Wednesday, July 12, 2023 we offer a guided city tour of the historic city center with an experienced guide (Graz Tourismus). A visit of Schloßberg is included (ride up and down with Schloßbergbahn and lift, and a drink at Aiola Upstairs). The tour meets in front of the Graz city hall at Hauptplatz 1 (at the arcade) and goes from 3:00 p.m. to 6:00 p.m.

At 7:00 p.m. we meet for the conference dinner. Further details on the conference dinner will be announced.

Restaurants located near the conference venue

Al Pomodoro, Heinrichstraße 45
Bierbaron, Heinrichstraße 56
Café Harrach, Harrachgasse 26
Cafeteria Resowi, Universitätsstraße 15
Das Liebig, Liebiggasse 2
Galliano, Harrachgasse 22
Gasthaus zum weißen Kreuz, Heinrichstraße 67
Lokal Müller, Villefortgasse 3
Mensa, Sonnenfelsplatz 1
Parks, Zinzendorfgasse 4
Posaune, Zinzendorfgasse 34
Propeller, Zinzendorfgasse 17
Sakana, Halbärthgasse 14
Unicafe, Heinrichstraße 36
Zeppelin, Goethestraße 21
Zu den 3 goldenen Kugeln, Heinrichstraße 18

Pharmacies located near the university

Glacis-Apotheke
Glacisstraße 31, 8010 Graz

Apotheke zu Maria Trost
Mariatrosterstraße 31, 8043 Graz

Apotheke zur göttlichen Vorsehung
Heinrichstraße 3, 8010 Graz

Emergency telephone numbers

Fire brigade: 122
Police: 133
Doctors' emergence service: 141
Ambulance: 144

UNI CAFE

VEGAN

Menu of the week

	SALAD BOWL	SUMMER SPECIAL	PASTA(A)	DAILY DISH
Mon. 10.7.	**Couscous** (Tomatoes/Cucumber/ Onion/Chickpeas/Peppers) (A) +Hummus +Avocado +Sheep Cheese +Goat Cheese	**Greek Salad Bowl** (A,G) Cucumber, Tomatoes, Feta, Olive Sticks	**With Basil Pesto** +Parmesan (G)	**Spinach-Chickpeas-Curry** With Rice & Lime
Tue. 11.7.	**Couscous** (Tomatoes/Cucumber/ Onion/Chickpeas/Peppers) (A) +Hummus +Avocado +Sheep Cheese +Goat Cheese	**Greek Salad Bowl** (A,G) Cucumber, Tomatoes, Feta, Olive Sticks	**With Basil Pesto** +Parmesan (G)	**Chili without meat** With Potato Bread (A) +Sour Cream (G) +0.50 €
Wed. 12.7.	**Couscous** (Tomatoes/Cucumber/ Onion/Chickpeas/Peppers) (A) +Hummus +Avocado +Sheep Cheese +Goat Cheese	**Greek Salad Bowl** (A,G) Cucumber, Tomatoes, Feta, Olive Sticks	**With Basil Pesto** +Parmesan (G)	**Lasagna Bolognese (Veggie)** (A,F,G,L)
Thu. 13.7.	**Couscous** (Tomatoes/Cucumber/ Onion/Chickpeas/Peppers) (A) +Hummus +Avocado +Sheep Cheese +Goat Cheese	**Greek Salad Bowl** (A,G) Cucumber, Tomatoes, Feta, Olive Sticks	**With Basil Pesto** +Parmesan (G)	**Greek Bean Stew** With Potato Bread (A,P) +Sheep Cheese
Fri. 14.7.	**Couscous** (Tomatoes/Cucumber/ Onion/Chickpeas/Peppers) (A) +Hummus +Avocado +Sheep Cheese +Goat Cheese	**Greek Salad Bowl** (A,G) Cucumber, Tomatoes, Feta, Olive Sticks	**With Basil Pesto** +Parmesan (G)	**Spinach-Ricotta-Lasagna** (A,C,G)

Salad Bowl 9.60 € / Summer Special 10.60 € / Pasta 9.60 € / Daily Dish 10.60 €

+Avocado 2 € / +Goat Cheese 2 €

A Cereals containing gluten • B Crustaceans • C Egg • D Fish • E Peanut • F Soy • G Milk or lactose
H Nuts • L Celery • M Mustard • N Sesame • O Sulphites • P Lupins • R Mollucs

Student Research Projects

In the summer semester 2023 there was a course *Topological Methods in Commutative Ring Theory* for PhD and Master students, organized jointly by V. Fadinger, C. Finocchiaro, D. Windisch at TU Graz and the University of Catania. As part of the course, students worked out small research projects in groups, that will be presented at the conference.

Research project 1

Marco Cottone (Catania), Balint Rago (Graz) und Giovanni Secreti (Catania) investigated phenomena in the paper by Maria Contessa, *On PM-rings*, Communications in Algebra **10** (1982), no. 1, 93-108. Theorem 2.1 of this work establishes a bijection between maximal ideals in a product of commutative rings and ultrafilters of Zariski closed subsets of the disjoint union of the sets of maximal ideals of these rings. Among other things, the students showed that this bijection induces a homeomorphism of the maximal spectrum of the product and the Wallman compactification of the disjoint union above.

Research project 2

Valentin Havlovec (Graz) and Mara Pompili (Graz) worked on the model theory of commutative rings using machinery of set-theoretical and topological flavor. They analyzed for several relevant ring theoretical properties whether they can be formulated as statements in the first-order language of rings. They applied the ultraproduct construction and some deep results from model theory, such as the Keisler–Shelah Theorem.

ABSTRACTS

Study of multiplicative derivation and its additivity

Wasim Ahmed

In this paper, we modify the result of M. N. Daif on multiplicative derivations in rings. He showed that the multiplicative derivation is additive by imposing certain conditions on the ring R. Here, we have proved the above result with lesser conditions than M. N. Daif for getting multiplicative derivation to be additive.

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On a group of vector-polynomial permutations

Amr Ali Abdulkader Al-Maktry

Let R be a finite commutative ring with identity $1 \neq 0$. A function $F : R^n \longrightarrow R$ is called a polynomial function on R if there exists a polynomial $f \in R[x_1, \ldots, x_n]$ such that $F(\vec{r}) = f(\vec{r})$ for each $\vec{r} \in R^n$. The set $\mathcal{F}(R^n)$ consisting of all polynomial functions is a finite commutative ring. Its group of units $\mathcal{F}(R^n)^\times$ is the subset of all polynomial functions that map R^n into $R^\times$. An n-vector function $\vec{F} : R^n \longrightarrow R^n$ is a vector-polynomial permutation of R^n if and only if:

(1) $\vec{F}$ permutes the elements of R^n;

(2) there exist polynomials $f_1, \ldots, f_n \in R[x_1, \ldots, x_n]$ such that

$$\vec{F}(\vec{r}) = \begin{pmatrix} f_1(\vec{r}) \\ f_2(\vec{r}) \\ \vdots \\ f_n(\vec{r}) \end{pmatrix}, \text{ for every } \vec{r} \in R^n.$$

For $n > 1$, we construct a group $\mathcal{A}_n$ of n-vector polynomial permutations of R^n. We show that the structure of $\mathcal{A}_n$ depends on the set of polynomial functions $\mathcal{F}(R^k)$ for $k = 1, \ldots, n-1$. More precisely, we have by iteration,

$$\begin{aligned} \mathcal{A}_n &= (\mathcal{F}(R^{n-1}) \rtimes \mathcal{F}(R^{n-1})^\times) \rtimes \mathcal{A}_{n-1} = \cdots = \\ &= \big((\mathcal{F}(R^{n-1}) \rtimes \mathcal{F}(R^{n-1})^\times) \rtimes \cdots \rtimes (\mathcal{F}(R) \rtimes \mathcal{F}(R)^\times) \rtimes \mathcal{P}(R)\big), \end{aligned}$$

where $\mathcal{P}(R)$ stands for the group of polynomial permutations of R. Further, we find the direct limit of the directed system $\{\mathcal{A}_n\}_{n \geq 2}$.

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Torsion-free modules over commutative domains of Krull dimension 1

Román Álvarez Arias

Let R be a commutative domain. Let $\mathcal{F}$ be the class of R-modules that are infinite direct sums of finitely generated torsion-free modules. In the talk we will discuss the question whether $\mathcal{F}$ is closed under direct summands. When R is a commutative local domain of Krull dimension 1, $\mathcal{F}$ being closed under direct summands is equivalent to say that any indecomposable, finitely generated torsion-free module has local endomorphism ring. For the global case in Krull dimension 1, we will also show that this property on $\mathcal{F}$ is inhereted when localizing at a maximal ideal and an interesting relation between the ranks of indecomposable modules over such localizations. The procedures we use to prove these results are based on generalizations to h-local domains of Package Deal Theorems from [1]. We will also discuss the property of being locally isomorphic (or locally a direct summand) versus being isomorphic (or a direct summand) in the setting of our problem.

The talk is based on a joint work with Dolors Herbera (Universitat Autònoma de Barcelona) and Pavel Příhoda (Charles University, Prague).

References

[1] L.S. Levy, C.J. Odenthal, *Package deal theorems and splitting orders in dimension 1*, Trans. Amer. Math. Soc. **348** (1996), 3457-3503.

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The v-number of Binomial Edge Ideals

Siddhi Balu Ambhore

The invariant v-number was introduced very recently in the study of Reed-Muller-type codes. Jaramillo and Villarreal started the study of the v-number of edge ideals. Inspired by their work in [1], we have initiated the study of the v-number of binomial edge ideals. We have studied some properties and the bounds of the v-number of binomial edge ideals. We have explicitly found the v-number of binomial edge ideals locally at the associated prime ideal corresponding to the cutset $\emptyset$. We have shown that the v-number of Knutson binomial edge ideals is less than or equal to the v-number of their initial ideals. We have classified all binomial edge ideals whose v-number is 1. We have further tried to relate the v-number with the Castelnuovo-Mumford regularity of binomial edge ideals and have given a conjecture in this direction.
This is a joint work with Kamalesh Saha and Indranath Sengupta.

References

[1] D. Jaramillo, R.H. Villarreal, *The v-number of edge ideals*, J. Combin. Theory Ser. A **177** (2021), 105310, 35 pp.

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Arithmetics of Flatness for Monoids

Gerhard Angermüller

Flatness in categories of modules over rings has been transferred to categories of acts over semigroups by Stenström in 1971. Focussing on commutative cancellative semigroups with unit ("monoids") we will show in this talk that this notion is useful too in considering arithmetical questions. Let D be a overmonoid of H, i.e. $H \subseteq D \subseteq q(H)$, where $q(H)$ is the quotient group of H.

- If D is a flat H-act, then there is a submonoid T of H such that $T^{-1}H = D$.

In contrast to this result, let R be a Dedekind domain whose class group is not a torsion group; then there is a flat overdomain of R, which is not a ring of fractions of R with respect to a multiplicatively closed subset of R.
Let R be a domain, M a torsion-free R-module and put $R^\bullet := R \setminus \{0\}$, $M^\bullet := M \setminus \{0\}$.

- M is a factorable R-module if and only if $M^\bullet$ is a flat $R^\bullet$-act and M is atomic.
- If M is a pre-Schreier R-module, then $M^\bullet$ is a flat $R^\bullet$-act; conversely, if R is a pre-Schreier domain and $M^\bullet$ is a flat $R^\bullet$-act, then M is a pre-Schreier R-module.
- If R is a pre-Schreier domain and M a flat R-module, then $M^\bullet$ is a flat $R^\bullet$-act.

There are factorial domains R and R-modules M such that $M^\bullet$ is a flat $R^\bullet$-act, but M is not a flat R-module.

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Representations of Leavitt path algebras

Pham Ngoc Ánh

First, we represent a Leavitt path algebra $L_K(E)$ of a digraph E over a field K as a quotient ring of a quiver algebra KE in Utumi's sense. Hence $L_K(E)$ is a perfect Gabriel localization of KE for finite digraph. This allows to connect representations of $L_K(E)$ to ones of KE with remarkable applications to prime factorizations in KE and irreducible representations of $L_K(E)$. Consequently, one has irreducible representations of $L_K(E)$ with non-commutative endomorphism rings and a room for see the role of infinite emitters in the study of Leavitt path algebras.

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Inverse semigroups of separated graphs and their associated algebras

Pere Ara

A separated graph is a pair (E, C), where E is a directed graph, $C = \bigsqcup_{v \in E^0} C_v$, and C_v is a partition of $s^{-1}(v)$ (into pairwise disjoint nonempty subsets) for every vertex v. For each separated graph (E, C), we will introduce its inverse semigroup $\mathcal{S}(E, C)$ and we will give a normal form for its elements. We will relate this inverse semigroup with some tame algebras associated to (E, C), such as the tame Cohn path algebra $C^{\mathrm{ab}}(E, C)$ and the tame Leavitt path algebra $L^{\mathrm{ab}}(E, C)$.
This is joint work in progress with Alcides Buss and Ado Dalla Costa, both from Universidade Federal de Santa Catarina.

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Multifurcus Semigroups and Applications to Classical Number Theory

Paul Baginski

A commutative cancellative semigroup S is bifurcus if, for every reducible element $s \in S$, there exist irreducible $a_1, a_2 \in S$ such that $s = a_1 a_2$. More generally, S is m-furcus (for $m \geq 2$) if for every nonunit $s \in S$, there exist irreducible $a_1, a_2, \ldots, a_k \in S$, with $k \leq m$ such that $s = a_1 a_2 \cdots a_k$. These semigroups are rather far from classical objects studied in nonunique factorization theory, such as algebraic number rings and polynomial rings. However, in this talk, we will show that such monoids occur rather frequently as subsemigroups of free abelian monoids of infinite rank, where mild hypotheses guarantee a semigroup with "multifurcus" factorization structure. Lastly, we show applications to several classes of semigroups arising naturally from classic number-theoretic functions.

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Global dimensions of ghor algebras and the Krull dimensions of their centers

CHARLIE BEIL

A ghor algebra is a special kind of noncommutative algebra that is constructed from an oriented graph (a dimer quiver) that embeds in a compact surface, with relations that identify homologous paths. The center R of such an algebra A is in general a non-noetherian coordinate ring of an affine toric variety with a 'positive dimensional point'. I will describe relationships between the global dimensions of the localizations of A at each maximal ideal of R, the Krull dimension of R, the dimension of the toric variety, and the genus of the surface the graph is embedded in. This is joint work with Karin Baur.

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Some remarks on additive and multiplicative decompositions of polynomials over unique factorization domains

Rachid Boumahdi

Let $\mathbb{F}_q$ denote the finite field of q elements, let f and g be monic polynomials over $\mathbb{F}_q$, and consider the polynomials $f * g$ and $f \circ g$ defined by: $f * g = \prod_\alpha \prod_\beta (x - (\alpha + \beta))$, $f \circ g = \prod_\alpha \prod_\beta (x - (\alpha\beta))$, where the product on the right is taken over all roots α and β of f and g respectively (including multiplicities), in the algebraic closure of $\mathbb{F}_q$. Although the roots of f and g may lie outside of $\mathbb{F}_q$, the polynomials $f * g$, $f \circ g$ have coefficients in $\mathbb{F}_q$. The operations $*$ and $\circ$ are binary operations on the set of monic polynomials over $\mathbb{F}_q$, called composed addition and composed multiplication. Additive decompositions over finite fields were widely studied by Brawley and Carlitz [2].

In this talk, we study the analogous notions of composed addition and results obtained in [1] (joint work with L. Benferhat et al.) and we give some remarks on the composed multiplication of two polynomials over unique factorization domains.

References

[1] L. Benferhat, M. Benoumhani, R. Boumahdi, J. Larone, *Additive Decompositions of Polynomials over Unique Factorization Domains*, J. Algebra Appl. **19** (2020), no. 8, 2050150, 11 pp.

[2] J.V. Brawley, L. Carlitz, *Irreducibles and the composed product for polynomials over a finite field*, Discrete Math. **65** (1987), no. 2, 115-139.

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Adequacy of matrices over commutative principal ideal domains

Victor Bovdi

The concept of a commutative adequate Bézout ring was introduced by Helmer [3] as a formalization of the properties of the entire analytic functions rings. Each commutative PID is adequate, but the converse is not true. Each adequate ring is an elementary divisor ring, but not every elementary divisor ring is adequate. Properties of adequate rings were investigated in [2, 3, 4, 5].

For non commutative Bézout rings we propose (see [1]):

Let K be a Bézout ring with $1 \neq 0$ and let $a \in K$. An element $b \in K$ is called a *left adequate* to $a \in K$ if there exist $s, t \in K \setminus U(K)$ such that $b = st$ and the following conditions hold:

(i) $s'K + aK \neq K$ for each $s' \in K \setminus U(K)$ such that $sK \subset s'K \neq K$;

(ii) for each $t' \in K \setminus U(K)$ such that $tK \subset t'K \neq K$ there exists a decomposition $st' = pq$ such that $pK + aK = K$.

The element s is called a *left adequate part* of b with respect to a. The right adequate part of b with respect to a is defined by analogy. A subset $A \subseteq K$ is called *left (right) adequate* if each of its elements is left (right) adequate to all elements of A. If each element of A is left and right adequate to the rest of elements, then the set A is called *adequate*.

`Theorem.`(see [1]) Let R be a commutative principal ideal domain such that $1 \neq 0$. The set of nonsingular matrices in $R^{2\times 2}$ is an adequate set.

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Numerical Semigroups and Music

Maria Bras-Amorós

We will elaborate on the algebraic structure of the sequence of harmonics when combined with equal temperaments. Fractals and the golden ratio appear surprisingly on the way. The sequence of physical harmonics is an increasingly enumerable submonoid of $(\mathbb{R}^+, +)$ whose pairs of consecutive terms get arbitrarily close as they grow. These properties suggest the definition of a new mathematical object which we denote a tempered monoid. Mapping the elements of the tempered monoid of physical harmonics from $\mathbb{R}$ to $\mathbb{N}$ may be considered tantamount to defining equal temperaments. The number of equal parts of the octave in an equal temperament corresponds to the multiplicity of the related numerical semigroup. Analyzing the sequence of musical harmonics we will derive two important properties that tempered monoids may have: that of being product-compatible and that of being fractal. We will demonstrate that, up to normalization, there is only one product-compatible tempered monoid, which is the logarithmic monoid, and there is only one nonbisectional fractal monoid which is generated by the golden ratio. The example of half-closed cylindrical pipes imposes a third property to the sequence of musical harmonics, the so-called odd-filterability property. We will prove that the maximum number of equal divisions of the octave such that the discretizations of the golden fractal monoid and the logarithmic monoid coincide, and such that the discretization is odd-filterable is 12. This is nothing else but the number of equal divisions of the octave in classical Western music.

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Some remarks on Prüfer rings with zero-divisors

Federico Campanini

We shall discuss some results about Prüfer rings with zero-divisors. We investigate the so-called "Prüfer-like conditions" (see [4] and the references therein) in several constructions, most of them related to pullbacks. It is well known that fiber products provide a rich source of examples and counterexamples in Commutative Algebra, because of their ability of producing rings with certain predetermined properties. Our investigation moves from very natural settings, for example those of regular conductor squares, up to more technical constructions, such as amalgamated and bi-amalgamated algebras. We also investigate Prüfer rings from other points of view. We introduce the notion of regular morphism and we prove that if a ring R is the homomorphic image of a Prüfer ring via a regular morphism, then R is Prüfer. Finally, we turn our attention to the ideal-theory of pre-Prüfer rings, proving a number of generalizations of some results of Boisen and Sheldon [5].

The talk is based on joint work with Carmelo Antonio Finocchiaro [1, 2, 3].

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Prime Factorization of ideals in commutative rings, with a focus on Krull rings

Gyu Whan Chang

Let R be a commutative ring with identity. The structure theorem says that R is a PIR (resp., UFR, general ZPI-ring, π-ring) if and only if R is a finite direct product of PIDs (resp., UFDs, Dedekind domains, π-domains) and special primary rings. All of these four types of integral domains are Krull domains, so motivated by the structure theorem, we study the prime factorization of ideals in a ring that is a finite direct product of Krull domains and special primary rings. Such a ring will be called a general Krull ring. It is known that Krull domains can be characterized by the star operations v or t as follows: An integral domain R is a Krull domain if and only if every nonzero proper principal ideal of R can be written as a finite v- or t-product of prime ideals. However, this is not true for general Krull rings. In this talk, we introduce a new star operation u on R, so that R is a general Krull ring if and only if every proper principal ideal of R can be written as a finite u-product of prime ideals. We also study several ring-theoretic properties of general Krull rings including Kaplansky-type theorem, Mori-Nagata theorem, Nagata ring, and u-almost Dedekind ring property. Finally, we note Juett's general w-ZPI ring.
All of the results in this talk are based on my recent paper [G.W. Chang and J.S. Oh, Prime factorization of ideals in commutative rings, with a focus on Krull rings. J. Korean Math. Soc. 60 (2023), no. 2, 407–464].

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The importance of being prime – a tribute to the work of Nick Baeth

Scott Chapman

"The importance of being prime: a nontrivial generalization for nonunique factorizations," will appear during 2023 in the *American Mathematical Monthly* and is the last paper co-authored by Nick Baeth. Using this paper as a backdrop, I will review Nick's scholarly works. While his life was all too brief, I will cite several examples which indicate that his influence within the community of those who work on problems involving nonunique factorizations will endure well after his passing.

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Factorization in certain integral domains and their ideal monoids

Hyun Seung Choi

We compute algebraic invariants concerning the factorization of one-dimensional Noetherian domains with nonzero conductor to its integral closure, focusing on numerical semigroup rings and Bass rings. The results corresponding to the monoid of ideals induced by these domains will be also presented.

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On some classes of subalgebras of Leavitt path algebras

Anna Cichocka

The history of Leavitt path algebras dates back to the 1960s, when William G. Leavitt posed a question regarding the existence of R rings that satisfy the equality $R^i = R^j$ as right-hand modules over R. The concept of Leavitt path algebras was introduced between 2005 and 2007. This construction was developed to algebraically represent combinatorial objects associated with Cuntz-Krieger algebras and C^*-algebras. The study of Leavitt path algebras is of interest to both algebraists and functional analysts, particularly those working with C^*-algebras. The flexible nature of Leavitt path algebra construction allows for the generation of numerous examples of algebras with specific properties.

An important special case of Leavitt path algebras is the class of matrix algebras $M_n(F)$ over a field F, where n is a natural number or infinity (infinite-size matrices have a countable number of rows and columns with a finite number of non-zero elements in each). In my presentation, I will recall definition of Leavitt path algebras. Based on this, I will demonstrate a construction of maximal commutative subalgebras of Leavitt path algebras and their connections with well-known examples of maximal commutative subalgebras of matrix algebras over a field.

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Arithmetic of minimal factorizations

Laura Cossu

Fundamental aspects of the classical theory of factorization can be widely generalized by combining the languages of monoids and preorders. Defining a suitable preorder on a monoid H permits to consider decompositions of its elements into arbitrary factors. In addition, an appropriate preorder on the free monoid over H allows the introduction of *minimal* factorizations, a refinement of classical factorizations that counter the blow-up phenomena which are typical of (atomic) factorizations in noncommutative or noncancellative monoids. In this talk we introduce the aforementioned notions, define some arithmetic invariants of minimal factorizations (including *minimal elasticity*) and present some finiteness results for such invariants. This is joint work with S. Tringali.

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Half-factoriality and length factoriality in monoids and domains

Jim Coykendall

In the study of factorization in integral domains, the “best” objects are unique factorization domains (UFDs). Besides the niceness of having each nonzero nonunit factor uniquely into irreducibles, UFDs have a less obvious characterization: a domain is a UFD if and only if every nonzero prime ideal contains a nonzero principal prime ideal.
In this talk, we will concentrate on factorization properties that are arguably the heirs apparent of the unique factorization property: the half-factorial and length factorial properties. We recall that a half-factorial domain (HFD) is an atomic integral domain with the property that if we have the irreducible factorizations

$$\alpha_1\alpha_2\cdots\alpha_m = \beta_1\beta_2\cdots\beta_n$$

then $m = n$. We also say that the atomic domain R is *length factorial* if any nonzero nonunit of R has (at most) one factorization of any length n (up to associates). Neither of these properties has any known universal ideal-theoretic characterization, so there is a wider variety of behaviors that these types of domains can exhibit. It is interesting to note that the strongest results about HFDs seem to occur in rings of algebraic integers where there is indeed an ideal-theoretic characterization of the half-factorial property.
In the absence of the strength of ideal-theoretic methods, there are a number of tools and results that have been developed to deal with these types of domains. It is our aim in this talk to discuss, as exhaustively as possible, the half-factorial and length factorial properties. We will review some history of the genesis of these properties, explore motivating examples, examine some relevant tools and techniques, and present some recent results concerning domains and monoids with these factorization structures.

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On the Apéry algorithm for a plane singularity

Marco D'Anna

A classical tool to get numerical invariants of a curve singularity is the study of its value semigroup. In case of a one branch singularity this semigroup is a numerical semigroup (i.e. a submonoid of $\mathbb{N}$ with finite complement in it); in case the singularity has h branches, this semigroup is a subsemigroup of $\mathbb{N}^h$, belonging to the class of the so-called "good semigroups". Despite their name, the combinatoric of good semigroups is quite problematic; moreover, for $h \geq 2$, it is an open problem to understand which good semigroups can be realized as value semigroups.
In case of a plane singularity with one branch, an old result of Apéry shows that there is a particularly strict connection between the value semigroups of the singularity and of its blowup; this connection is obtained using a particular set of generators of the semigroup, named "Apéry set". In fact, using that result, it is possible to show very easily, that the equisingularity classes given by the multiplicity sequence and by the value semigroup coincide. In particular, this method allows to reconstruct the numerical semigroup associated to a plane branch singularity starting from the multiplicity sequence.
When the singularity has more than one branch, in order to generalize the Apéry result, two main problems arise: firstly, the Apéry set becomes an infinite set; secondly, in the process of blowing-up it is necessary to deal with semilocal rings, that cannot be presented as quotients of a power series ring in two variables, as it happens in the local case. These problems where partially solved twenty years ago in the two branch case, but the general case is still open.
In my talk, after describing some key definitions and results on value semigroups and good semigroups of a curve singularity with h branches, I will explain explaining the Apéry process for a plane branch and then, I will present some recent results obtained in a joint project with F. Delgado de la Mata, L. Guerrieri, N. Maugeri and V. Micale, that are a significant progress toward a complete solution of this problem.

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Rings whose associated extended zero-divisor graphs are complemented

Brahim El Alaoui

Let R be a commutative ring with identity $1 \neq 0$. The extended zero-divisor graph, denoted by $\overline{\Gamma}(R)$, is the simple graph with vertex set is the set of nonzero zero divisors and two distinct vertices x and y are adjacent if and only if $x^n y^m = 0$ with $x^n \neq 0$ and $y^m \neq 0$ for some integers $n, m \in \mathbb{N}^*$. The aim of this talk is to present when $\overline{\Gamma}(R)$ is complemented or uniquely complemented. We also give a complete characterization of when $\overline{\Gamma}(R)$ of a finite ring is complemented. Finally, using the direct product of rings and idealizations of modules, we present some examples of extended zero-divisor graphs which are complemented.

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On Associative formal power series

Susan El-Deken

A power series $F(x,y) \in R[\![x,y]\!]$ is called associative if $F(F(x,y),z)) = F(x,F(y,z)) \in R[\![x,y,z]\!]$, and $\operatorname{ord}(F(x,y)) \geq 1$. A formal group over R is an associative power series $F(x,y) \in R[\![x,y]\!]$ satisfying $F(x,y) \equiv x+y \mod M(x,y)^2$.

Fripertinger et al. [1] investigated associative power series which are not formal groups and obtained a complete characterization of them over arbitrary fields. Franz Halter-Koch [3] characterized associative power series in two indeterminates over a commutative ring. In this work we shall give the characterization of associative power series in two indeterminates over a non-commutative ring.

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The t-closure and w-closure semistar operations on a commutative ring

Jesse Elliott

We present definitions and equivalent characterizations of the t-closure and w-closure semistar operations on a commutative ring, as well as some of their many applications to the study of commutative rings. For example, we show how to use these operations to characterize the Krull rings, PVMRs, and Manis valuation rings. We also present some open problems regarding these classes of rings.

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Pre-Lie Algebras

Alberto Facchini

We will present some notions concerning pre-Lie algebras and some applications of the notions of pre-morphism and pre-derivation (Lie derivations) for arbitrary non-associative algebras over a commutative ring k with identity. We will consider both pre-Lie k-algebras and Lie-admissible k-algebras. Associating with any algebra $(A, \cdot)$ its sub-adjacent anticommutative algebra $(A, [-, -])$ is a functor from the category of k-algebras with pre-morphisms to the category of anticommutative k-algebras. Pre-morphisms can be dualized in various ways to generalized morphisms (related to pre-Jordan algebras) and anti-pre-morphisms (related to anti-pre-Lie algebras). We consider idempotent pre-endomorphisms (generalized endomorphisms, anti-pre-endomorphisms). Idempotent pre-endomorphisms are related to semidirect-product decompositions of the sub-adjacent anticommutative algebra.

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Prescribed lengths of factorizations of integer-valued polynomials on Krull domains with prime elements

Victor Fadinger

In 2013 Frisch proved that for all positive integers k and $1 < n_1 \leq \ldots \leq n_k$ there exists an integer-valued polynomial on $\mathbb{Z}$ which has precisely k essentially different factorizations into irreducible elements, whose lengths are exactly $n_1, \ldots, n_k$. In 2019 Frisch, Nakato and Rissner generalized this result, replacing $\mathbb{Z}$ by any Dedekind domain with infinitely many maximal ideals, each of them of finite index. The semilocal case remained open. We prove an analogous result in case D is a Krull domain with a prime element p such that D/pD is finite. From this we deduce a complete description of the system of sets of lengths of integer-valued polynomials on unique factorization domains, so in particular on semilocal Dedekind domains.
This is joint work with Daniel Windisch and Sophie Frisch.

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On invertible and divisorial ideals of $\mathrm{Int}(D)$

Carmelo Antonio Finocchiaro

Let D be a Dedekind domain with finite residue fields and let K be the quotient field of D. Let

$$\mathrm{Int}(D) := \{f(T) \in K[T] \mid f(D) \subseteq D\}$$

denote the ring of integer-valued polynomials over D. The aim of this talk, based on a paper written jointly with A. Loper, is to describe a topological perspective on the study of invertible and divisorial ideals of $\mathrm{Int}(D)$. Using topology is particularly helpful to provide natural examples of divisorial ideals of $\mathrm{Int}(\mathbb{Z})$ that are not finitely generated.

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On p-Frobenius of affine semigroups

J. Ignacio García-García

An affine semigroup $S \subseteq \mathbb{N}^q$ is a set containing 0 and closed under addition. A finite set $A = \{a_1, \ldots, a_h\} \subseteq \mathbb{N}^q$ is a minimal generating set of S if every element of S can be expressed as a non-negative integer linear combination of the elements in A. The set $\mathrm{Z}_n(S)$ is the set of all $\lambda = (\lambda_1, \ldots, \lambda_h) \in \mathbb{N}^h$ such that $n = \sum_{i=1}^{h} \lambda_i a_i$ for a given generating set A, where n is an element of S. The minimum integer cone containing S is denoted by $\mathcal{C}(S)$.

If $q = 1$, S is a numerical semigroup if $\mathbb{N} \setminus S$ is finite ($\gcd(a_1, \ldots, a_h) = 1$). We assume $a_1 < \cdots < a_h$ and fix a monomial order $\preceq$ on $\mathbb{N}^q$ for further discussions. This setting allows us to relate affine semigroups with the theory of factorization in commutative monoid rings and thus generalize concepts such as the Frobenius number to affine semigroups. The Frobenius number/vector is an important invariant of semigroups. For numerical semigroups, it is defined as the largest integer that cannot be expressed as a positive linear combination of the minimal generators of the semigroup. For affine semigroups, the Frobenius vector is the maximum integer vector satisfying a similar condition, but it may not exist in general. When the minimum integer cone containing the semigroup is $\mathbb{N}^q$ and $\mathbb{N}^q \setminus S$ is finite, the maximum integer vector can be computed by setting $\max_{\preceq}(\mathcal{C} \setminus S)$, where $\preceq$ is a fixed monomial order ([2]). In [3], the possible Frobenius vectors for such affine semigroups S are studied.

In this work, inspired by the concept introduced in [1] we delve into the notion of p-Frobenius vector for an affine semigroup S. The p-Frobenius vector of S (with respect to the monomial order $\preceq$) is defined as $F_p(S) = \max_{\prec} n \in \mathcal{C}(S) \mid \sharp \mathrm{Z}_n(S) \leq p$.

The main goal of our work is to provide an algorithm for computing $F_p(S)$. Additionally, we significantly improve the computation of $F_1(S)$ and $F_2(S)$ by using presentations of the affine semigroup. Finally, we investigate some properties of the p-Frobenius number for numerical semigroups.

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The ideal class monoid of a numerical semigroup

Pedro A. García-Sánchez

Let S be a numerical semigroup (a submonoid of the set of nonnegative integers under addition such that $\max(\mathbb{Z} \setminus S)$ exists). A set I of integers is said to be an ideal of S if $I + S \subseteq I$ and I has a minimum. If I and J are ideals of S, we write $I \sim J$ if there exists an integer z such that $I = z + J$. The ideal class monoid of S is defined as the set of ideals of S modulo this relation. It can be easily proven that this monoid is isomorphic to the monoid of ideals of S having minimum equal to zero.
In this talk, we present new bounds for the cardinality of the ideal class monoid of a numerical semigroup, some of them obtained using the Kunz coordinates of the semigroup. The ideal class monoid of a numerical semigroup is a finite commutative monoid, which is reduced but not unit-cancellative, and can be endowed with two preorders (inclusion and the preorder induced by addition). We describe the notable elements of the resulting posets with respect to these preorders, showing that some recover classical invariants of the numerical semigroup. We also study what are the irreducible and prime elements with respect to addition, as well as quarks and atoms. It turns out that the ideal class monoid of a numerical semigroup might have no atoms at all, and that its idempotent quarks correspond with unitary extensions of the numerical semigroup. Actually, in this new family of monoids, the concepts of irreducible, prime, atom, and quark differ, giving examples where one can clearly differentiate them. We prove that a numerical semigroup is irreducible if and only if its ideal class monoid has at most two quarks, characterizing in this way a remarkable property of the semigroup (symmetry or pseudo-symmetry) in terms of the Hasse diagram of its ideal class group with respect to the preorder induced by addition.
We will present at the end of the talk a series of open questions.
This is a joint work with L. Casabella and M. D'Anna.

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Strong Types of Atomicity

Felix Gotti

A cancellative commutative monoid is called atomic if every nonunit element factors into atoms (i.e., irreducible elements), and an integral domain is called atomic if its multiplicative monoid is atomic. A significant part of the theory of non-unique factorizations takes place in the class of atomic monoids/domains, and most of the systematically studied factorization properties imply the ascending chain condition on principal ideals (ACCP), which is a condition stronger than that of being atomic. The primary purpose of this talk is to discuss various stronger types of atomicity, including strong atomicity (as introduced in [1]) and hereditary atomicity (as introduced in [2]), in the settings of commutative monoids and integral domains. We will also go into some known (and unknown) interconnections between the discussed types of atomicity and the ACCP.

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On radical ideals of noncommutative rings

Nico Groenewald

Let $J(R)$ denote the Jacobson radical of a commutative ring R. In [1] the notion of J-ideals were introduced. If $N(R)$ denotes the prime radical of a commutative ring then in [2] the notion of an N ideal of a commutative ring was introduced and studied. In this note we show that these results are special cases of a more general situation. We define $\rho-$ideals for a special radical ρ and prove that most of the results of the above mentioned papers are satisfied for non-commutative rings as a special case.

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Zero-Sums in p-groups via a Generalization of the Ax-Katz Theorem

David J. Grynkiewicz

Let G be a finite abelian group with exponent n. A classical topic in Combinatorial Number Theory is the study of conditions on a sequence S of terms from G that force S to have a nontrivial subsequence whose terms sum to zero (called a zero-sum sequence). For instance, the minimal length for the sequence which always ensures a nontrivial zero-sum subsequence is called the Davenport constant of G, denoted $\mathsf{D}(G)$. Its value remains open in general but is known for p-groups. As a variation, for an integer $k \geq 1$, let $\mathsf{s}_{kn}(G)$ denote the minimal length for the sequence which always ensures a zero-sum subsequence whose length is exactly kn. The behavior of this constant varies quite drastically depending on k. For $k = 1$, it can grow exponentially (with respect to the rank). In contrast, it is known that $\mathsf{s}_{kn}(G) = kn + \mathsf{D}(G) - 1$ once k is sufficiently large (specifically $k \geq |G|/n$), which for p-groups is a linear bound (with respect to the rank). In this talk, we will be concerned with how large k must be to achieve $\mathsf{s}_{kn}(G) = kn + \mathsf{D}(G) - 1$. It is conjectured that this occurs once $k \geq d$, where $d = \lceil \frac{\mathsf{D}(G)}{n} \rceil$. For p-groups, Xiaoyu He showed that $k \geq p + d$ suffices so long as p is sufficiently large (specifically, $p \geq d$), and he posed the problem of finding a sufficient bound on k depending only on d. Here, we answer that question by showing $k > \frac{d(d-1)}{2}$ is sufficient so long as p is sufficiently large (specifically, $p > d(d-1)$).

The work of Xiaoyu He and others relied heavily on an algebraic method developed by Ronyai, while work of Reiher for the case $k = 1$ when $d = 2$ rather used the Chevalley-Warning Theorem regarding the p-divisibility of a collection of multivariate polynomials over a finite field. The Ax-Katz Theorem generalizes the Chevalley-Warning Theorem by giving higher order p divisibility estimates for the number of commons roots. Using ideas of Wilson and Zhi-Wei Sun, one can give a weighted generalization of the prime field case of the Ax-Katz Theorem that considers the common roots of polynomials modulo *varying* prime powers, and it is this generalization, which combines features of Ronyai's Method and the Chevalley-Warning Theorem into a single result, that is used in the proof in combination with combinatorial reduction arguments. It also gives a simple proof of the exact value of the Davenport Constant for p-groups that avoids the use of group algebras.

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Non-integrally closed Kronecker function rings and integral domains with a unique minimal overring

Lorenzo Guerrieri

It is well-known that an integrally closed domain D can be express as the intersection of its valuation overrings but, if D is not a Prüfer domain, the most of valuation overrings of D cannot be seen as localizations of D. The Kronecker function ring of D is a classical construction of a Prüfer domain which is an overring of $D[t]$, and its localizations at prime ideals are of the form $V(t)$ where V runs through the valuation overrings of D. This fact can be generalized to arbitrary integral domains by expressing them as intersections of overrings which admit a unique minimal overring. In this talk we extend the definition of Kronecker function ring to the non-integrally closed setting by studying intersections of Nagata rings of the form $A(t)$ for A an integral domain admitting a unique minimal overring.

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Leavitt path algebras

Roozbeh Hazrat

Leavitt path algebras were introduced by Abrams-Aranda Pino and Ara-Moreno-Pardo, as certain path algebras associated to directed graphs, less than 20 years ago. We give a survey of the recent advances in the theory of Leavitt path algebras and the invariants for their classifications. These algebras have appeared in a variety of different contexts as diverse as analysis, symbolic dynamics, and representation theory.

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Cousin complexes via total fractions

I-Chiau Huang

Cousin complexes appeared in Grothendieck's theory of duality [1]. While looking for some sort of natural minimal injective resolution for a Gorenstein ring, R. Sharp also defined Cousin complexes [2]. Concrete Cousin complexes are rarely seen for explicitly given Noetherian rings except for easy cases. We have a new framework for Cousin complexes so that computations become feasible. Let R be a commutative Noetherian ring. For an R-module M, its Cousin complex

$$\cdots \bigoplus_{\text{height } \mathfrak{p} = r} \mathrm{H}^r_{\mathfrak{p}R_\mathfrak{p}}(M_\mathfrak{p}) \to \bigoplus_{\text{height } \mathfrak{q} = r+1} \mathrm{H}^{r+1}_{\mathfrak{q}R_\mathfrak{q}}(M_\mathfrak{q}) \to \cdots$$

is built up by local cohomology modules. Elements of top local cohomology modules can be described by generalized fractions. A fundamental and natural question is to describe $\mathrm{H}^r_{\mathfrak{p}R_\mathfrak{p}}(M_\mathfrak{p}) \to \mathrm{H}^{r+1}_{\mathfrak{q}R_\mathfrak{q}}(M_\mathfrak{q})$ in the coboundary map in terms of generalized fractions. For a height r prime $\mathfrak{p}$, a system of parameters $f_1, \dots, f_r$ of $\mathfrak{p}$ and $m \in M$, $f \in R \setminus \mathfrak{p}$, the expected formula

$$\begin{bmatrix} m/f \\ f_1, \dots, f_r \end{bmatrix} \mapsto \begin{bmatrix} m \\ f_1, \dots, f_r, f \end{bmatrix}$$

is not even well-defined, *cf.* [3]. To remedy the defect, we introduce new notions: total fractions and faithful representations. Here is an explicit formula occurring in the Cousin complex. Let κ be a field and $R := \kappa[\![X^4, X^3Y, XY^3, Y^4]\!]$. For the maximal ideal $\mathfrak{m}$ and the prime ideal $\mathfrak{p}$ generated by X^4 up to radical, the R-linear map $\mathrm{H}^1_{\mathfrak{p}R_\mathfrak{p}}(R_\mathfrak{p}) \mapsto \mathrm{H}^2_{\mathfrak{m}R_\mathfrak{m}}(R_\mathfrak{m})$ in our approach gives rise to

$$\begin{bmatrix} 1/Y^4 \\ X^3Y \end{bmatrix} \mapsto \begin{bmatrix} XY^3 \\ X^4, Y^8 \end{bmatrix}.$$

It is not clear how other approaches to Cousin complexes provide such a formula.

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Regular t-ideals of polynomial rings and semigroup rings with zero divisors

Jason Juett

For an integrally closed domain D with quotient field K, it is well known that every fractional t-ideal of $D[X]$ has the form $hI[X]$ with $h \in K(X)$ and I a t-ideal of D. In this talk, I will share some of my recent work that generalizes this useful result to certain polynomial rings and semigroup rings with zero divisors. Applications of this work include characterizing the semigroup rings with torsion-free cancellative monoids of exponents that are Prüfer v-multiplication rings or (generalized) greatest common divisor rings. This work can also be used to confirm a conjecture of Glaz (2000), namely that her (generalized) greatest common divisor ring property ascends to polynomial rings.

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A research on the generalizations of modules whose submodules are isomorphic to a direct summand

Fatih Karabacak

This is a joint work with Özgür Taşdemir. In 2018, Behboodi and his colleagues introduced and investigate a new module family which called virtually semisimple modules: A module M is called virtually semisimple if every submodule of M is isomorphic to a direct summand of M. Later, this new module family and related concepts were investigated by many authors. For instance, in 2022, Karabacak and his colleagues introduced virtually extending modules which is a generalization of both extending modules and virtually semisimple modules: They call a module M is virtually extending if every complement submodule of M is isomorphic to a direct summand of M. In this study, we present some new results on some generalizations of virtually semisimple modules.

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Banach algebras of convolution type operators with piecewise quasicontinuous data

Yuri Karlovich

Given $p \in (1, \infty)$ and a Muckenhoupt weight $w \in A_p(\mathbb{R})$, let $\mathcal{B}_{p,w}$ be the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$. Let $PQC_{\mathbb{R}}$ be the C^*-algebra all piecewise quasicontinuous functions on the real line $\mathbb{R}$ that are equivalent to piecewise slowly oscillating functions at infinity and let $PQC_{\mathbb{R},p,w}$ be the corresponding Banach algebra of Fourier multipliers on the space $L^p(\mathbb{R}, w)$. For a subclass of weights w, the Banach algebras $\mathfrak{A}_{p,w} \subset \mathcal{B}_{p,w}$ generated by all multiplication operators aI with $a \in PQC_{\mathbb{R}}$ and by all convolution operators $W^0(b)$ with $b \in PQC_{\mathbb{R},p,w}$ are studied. First, a Fredholm symbol calculus is constructed for the Banach algebra $\mathcal{Z}_{p,w} \subset \mathfrak{A}_{p,w}$ generated by the operators $aW^0(b)$ with quasicontinuous data functions $a \in QC_{\mathbb{R}}$ and $b \in QC_{\mathbb{R},p,w}$ that are equivalent to some slowly oscillating functions at infinity. Then a Fredholm symbol calculus for the Banach algebra $\mathfrak{A}_{p,w}$ is constructed and a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ is established by applying the Allan-Douglas local principle, limit operators techniques and the two idempotents theorem. Investigations of sums of ideals is crucial in the study. The talk is partially based on a joint work with C.A. Fernandes and A.Yu. Karlovich.

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On Waring numbers of henselian rings

Tomasz Kowalczyk

Let $n > 1$ be a positive integer and R be a commutative ring with unity. We define the nth Waring number $w_n(R)$ as the smallest positive integer g such that every sum of nth powers of elements of R can be written as a sum of at most g nth powers of elements of R.

Let R be a henselian local ring with residue field k of nth level $s_n(k)$. We give some upper and lower bounds for the nth Waring number $w_n(R)$ in terms of $w_n(k)$ and $s_n(k)$. In large number of cases we are able to compute $w_n(R)$. Similar results for the nth Waring number of the total ring of fractions of R are obtained. We then provide applications. In particular we compute $w_n(\mathbb{Z}_p)$ and $w_n(\mathbb{Q}_p)$ for $n \in \{3, 4, 5\}$ and any prime p.

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Automorphisms of Weil algebras

Miroslav Kureš

The *algebra of truncated polynomials* $\mathbb{D}_n^r$ is defined as the following quotient algebra of the polynomial algebra:

$$\mathbb{D}_n^r = \mathbb{R}[x_1, \ldots, x_n]/\langle x_1, \ldots, x_n\rangle^{r+1},$$

where $\langle x_1, \ldots, x_n\rangle$ is the ideal generated by monomials $x_1, \ldots, x_n$. (A prototypical example is the algebra of dual numbers $\mathbb{D}$, where $r = n = 1$.) Equivalence classes which are elements of $\mathbb{D}_n^r$ are expressed as polynomials of the degree at most r (for simplicity, again with indeterminates denoted by $x_1, \ldots, x_n$), when multiplying them, the powers from the $(r+1)$-th are zeroed. Hence every truncated polynomial has its constant part and nilpotent part.

Considering ideals $\mathfrak{j} \subseteq \langle x_1, \ldots, x_n\rangle^2$, by the *Weil algebra* we understand every $\mathbb{R}$-algebra A of the form

$$A = \mathbb{D}_n^r/\mathfrak{j}.$$

These algebras play an important role in differential geometry, above all they bijectively correspond to the product preserving functors from the category of manifolds to the category of fibered manifolds. (E.g. $\mathbb{D}$ corresponds to the tangent functor T.)

In the talk, we focus on the structure of automorphism groups $\operatorname{Aut} A$ of Weil algebras A. In particular, we present results about

(i) the connected identity component G_A of $\operatorname{Aut} A$ which is a crucial tool not only for understanding $\operatorname{Aut} A$, but also A itself;

(ii) the fixed points subalgebra

$$SA = \{u \in A; \alpha(u) = u \text{ for all } \alpha \in \operatorname{Aut} A\}.$$

We also demonstrate applications of these results in geometry.

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Representation and Subrepresentation of Γ-monoids

Flor de May Lañohan

This is a joint work with Jocelyn Vilela. The representation theory of finite groups was first presented by Frobenius in 1896. The goal of representation theory is to understand the algebraic structure of groups by transforming algebraic objects into matrices. Hazrat and Li first established the notion of Γ-monoid, which is a monoid with a group Γ acting on it. In this study, we explored the representation of Γ-monoids or more specifically, the subrepresentation of a representation. We first introduced the concept of the Γ-invariant to define and investigate the subrepresentation of a representation. Then we proved that a subrepresentation is also a representation and demonstrated that a restriction map to the kernel, image, and inverse image of a Γ-linear map are subrepresentations.

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On rational extension of modules

Gangyong Lee

The rational extension of a ring is developed by R.E. Johnson in 1951 and Y. Utumi in 1956 in order to generalize the quotients ring theory. In 1958, G.D. Findlay and J. Lambek defined a relationship between three R-modules, $A_R \leq B_R(C_R)$, which means that A_R is a relatively dense submodule in B_R to a R-module C_R in this talk. After that, R. Courter [2, 3] in 1966 and 1969 and S.H. Brown [1] in 1973 studied rationally complete modules by using the relatively dense property.
In this talk, we provide several new characterizations of the maximal right ring of quotients of a ring by using the relatively dense property. In addition, we obtain several properties of the rational hull of modules and rationally complete modules. For example, we show that the rational hull of a quasi-continuous or quasi-injective module is also quasi-continuous or quasi-injective, respectively. Also we prove that the rational hull of a polyform module is also quasi-injective. We obtain the equivalent condition for the finite direct sum of the rational hulls of submodules to be the rational hull of the direct sum of those submodules.

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The i-extended zero-divisor graphs of idealizations

Raja L'hamri

Let R be a commutative ring with zero-divisors $Z(R)$ and i be a positive integer. The i-extended zero-divisor graph of R, denoted by $\overline{\Gamma}_i(R)$, is the (simple) graph with vertex set $Z(R)^* = Z(R) \setminus \{0\}$, the set of nonzero zero-divisors of R, and two distinct nonzero zero-divisors x and y are adjacent whenever there exist two positive integers $n, m \leq i$ such that $x^n y^m = 0$ with $x^n \neq 0$ and $y^m \neq 0$. In this talk, we consider the i-extended zero-divisor graphs of idealizations $R \ltimes M$ (where M is an R-module). The aim of this talk is to study in detail the behaviour of the filtration $(\overline{\Gamma}_i(R \ltimes M))_{i \in \mathbb{N}^*}$ as well as the relations between its terms. We also characterize the girth and the diameter of $\overline{\Gamma}_i(R \ltimes M)$ and we give answers to several interesting and natural questions that arise in this context.

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Divisibility and a weak ascending chain condition on principal ideals

Bangzheng Li

An integral domain R is atomic if each nonzero nonunit of R factors into irreducibles. In addition, an integral domain R satisfies the ascending chain condition on principal ideals (ACCP) if every increasing sequence of principal ideals (under inclusion) becomes constant from one point on. Although every integral domain satisfying ACCP is atomic, examples of atomic domains that do not satisfy ACCP are notoriously hard to construct. The first of such examples was constructed by A. Grams back in 1974. In this talk we will discuss the class of atomic domains that do not satisfy ACCP. We will put special emphasis on integral domains satisfying the weak-ACCP, a condition weaker than the ACCP recently introduced in a joint paper with Felix Gotti.

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Interval rings

Alan Loper

Let D be a two-dimensional regular local ring with maximal ideal $m = (x, y)$. Choose a closed interval $[a, b]$ within the positive real numbers. For each real number in $[a, b]$ we define a collection of valuation domains centered on m. The intersection of these valuation domains is a one-dimensional local ring with a lot of interesting properties which we will explore.

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(σ, τ)-Derivations of Number Rings

Praveen Manju

For a commutative ring R with unity and for a pair of R-algebra endomorphisms (σ, τ) on an R-algebra $\mathcal{A}$, a (σ, τ)-derivation $D : \mathcal{A} \to \mathcal{A}$ is an R-linear map that satisfies the Leibniz rule: $D(\alpha\beta) = D(\alpha)\tau(\beta) + \sigma(\alpha)D(\beta)$ for all $\alpha, \beta \in \mathcal{A}$. It is called inner if there exists some $\gamma \in \mathcal{A}$ such that $D(\alpha) = \gamma\tau(\alpha) - \sigma(\alpha)\beta$ for all $\alpha \in \mathcal{A}$. In this article, we study (σ, τ)-derivations of number rings. A number ring is a subring of an algebraic number field and is a commutative unital $\mathbb{Z}$-algebra. First, we develop a necessary condition for an R-linear map D on $\mathcal{A}$ to be a (σ, τ)-derivation. We show via counterexamples that this condition, in general, is not sufficient. We further give a characterization for a (σ, τ)-derivation D on $\mathcal{A}$ to be inner. We apply the results obtained to study (σ, τ)- and inner (σ, τ)-derivations of number rings. We characterize all (σ, τ)- and inner (σ, τ)-derivations of the rings of algebraic integers of quadratic number fields. We characterize all (σ, τ)-derivations of the ring of algebraic integers $\mathbb{Z}[\zeta]$ of a p^{th}-cyclotomic number field $\mathbb{Q}(\zeta)$, where p is an odd rational prime and ζ is a primitive p^{th}-root of unity. We also conjecture a sufficient condition for a (σ, τ)-derivation D on $\mathbb{Z}[\zeta]$ to be inner. This is done with the help of software SAGE and MATLAB. We further characterize all (σ, τ)- and inner (σ, τ)-derivations of the bi-quadratic number ring $\mathbb{Z}[\sqrt{m}, \sqrt{n}]$, where m and n are distinct square-free rational integers. In the process, the ranks and an explicit basis of the (σ, τ)-derivation algebras of these number rings, when considered as $\mathbb{Z}$-modules are also determined. Finally, we pose some open questions.

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On the Smith normal form of dual integer matrices

Baraah Maya

Let A be a nonzero $m \times m$ matrix over a principal ideal domain R. There exist invertible $m \times m$ and $n \times n$ matrices P, Q with coefficients in R such that the product PAQ is the matrix:

$$S = \begin{pmatrix} \alpha_1 & 0 & 0 & & \dots & & 0 \\ 0 & \alpha_2 & 0 & & \dots & & 0 \\ 0 & 0 & \ddots & & & & \\ & & & \alpha_r & & & 0 \\ \vdots & & & & 0 & & \vdots \\ & & & & & \ddots & \\ 0 & & & & & & 0 \end{pmatrix}$$

and the diagonal elements α_i satisfy $\alpha_i | \alpha_{i+1}$ for all $1 \leq i \leq r$. The matrix S is the Smith normal form of the matrix A. The elements α_i are unique up to multiplication by a unit and called the elementary divisors. They can be computed as:

$$\alpha_i = \frac{d_i(A)}{d_{i+1}(A)}$$

where $d_i(A)$ equals the greatest common divisor of the determinants of all $i \times i$ minors of the matrix A and $d_0(A) = 1$. Let us consider the ring:

$$\mathbb{Z}[\varepsilon] = \{a + b\varepsilon; \quad a, b \in \mathbb{Z}\}$$

The ring $\mathbb{Z}[\varepsilon]$ is not a principal ideal domain. This study discusses how to define the operation of division in $\mathbb{Z}[\varepsilon]$, how to find the divisors of a dual integer, how to find the common divisors of two dual integers, how to find the greatest common divisor of two dual integers and the necessary and sufficient condition for the existence of the Smith normal form of a dual integer matrix.

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Davenport constants and related problems

Eshita Mazumdar

Zero-sum problems are basically combinatorial in nature. It deals with the condition that ensures that a given sequence over a finite group has a zero-sum subsequence with some prescribed properties. There are many invariants associated with zero-sum problems. One such invariant is the Davenport constant. The original motivation for introducing the Davenport constant was to study the problem of non-unique factorization domain over number fields. The precise value of this group invariant for any finite abelian group is still unknown. In this talk, I will introduce several exciting combinatorial problems related to the Davenport constant, such as an extreme problem related to the weighted Davenport constant,study the strong Davenport constant for finite groups, and so on. If time permits, I would also like to discuss my ongoing work.

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Graded Semigroups

Zachary Mesyan

This talk is based on joint work with Roozbeh Hazrat, where we systematically develop a theory of graded semigroups, that is, semigroups S partitioned by groups, in a manner compatible with the multiplication on S. We prove semigroup analogues of well-known theorems of Cohen/Montgomery and Dade about categories of graded rings, discuss graded semigroup Morita equivalence, give a graded Vagner-Preston theorem, and provide numerous examples of naturally-occurring graded semigroups. We also explore connections between graded semigroups, graded groupoids, and graded rings, including Leavitt path algebras.

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Unique Maximal Rings of Functions

Johan Meyer

For several classes of groups G we characterize when the near-ring $M_0(G)$ of 0-preserving selfmaps on G contains a unique maximal ring. Definitive results are obtained for finite abelian, finite nilpotent, and finite permutation groups. As an application we determine those finite groups G such that all rings in $M_0(G)$ are commutative.

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On Waring numbers of henselian rings

Piotr Miska

Let $n > 1$ be a positive integer and R be a commutative ring with unity. We define the nth Waring number $w_n(R)$ as the smallest positive integer g such that every sum of nth powers of elements of R can be written as a sum of at most g nth powers of elements of R.

Let R be a henselian local ring with residue field k of nth level $s_n(k)$. We give some upper and lower bounds for the nth Waring number $w_n(R)$ in terms of $w_n(k)$ and $s_n(k)$. In large number of cases we are able to compute $w_n(R)$. Similar results for the nth Waring number of the total ring of fractions of R are obtained. We then provide applications. In particular we compute $w_n(\mathbb{Z}_p)$ and $w_n(\mathbb{Q}_p)$ for $n \in \{3, 4, 5\}$ and any prime p.

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Row-factorization matrices and type of almost Gorenstein monomial curves

Alessio Moscariello

Almost Gorenstein rings are a class of Cohen-Macaulay rings, introduced by Barucci and Fröberg in 1997 in the context of analytically unramified rings of dimension one. It is known that, in this context, a local ring is Gorenstein if and only if it is Cohen-Macaulay and its Cohen-Macaulay type is equal to one; almost Gorenstein rings extend the class of Gorenstein rings to arbitrary type (almost Gorenstein rings of type one are Gorenstein). For one-dimensional analytically unramified rings, such as local rings associated to monomial curves, if the embedding dimension e is at least 4, the Cohen-Macaulay type t is not bounded. On the other hand, various studies asked whether, for an almost Gorenstein ring, t is bounded by a function of e.
In this talk we show how, using row-factorization matrices (matrices encoding the factorization properties of integers with respect to the value semigroup of the ring), some boundedness results can be obtained for $e \leq 5$, and make some remarks on the next case $e = 6$, which might be crucial for understanding the general case.

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On value semigroups associated to curves

Julio-José Moyano-Fernández

In this talk we will report about some recent results concerning value semigroups associated to algebraic curves. We will focus on the value semigroup at infinity associated to a curve having only one place at infinity, describing to which extent this is a good object to classify this kind of singularities. This is part of a joint project with C. Galindo, F. Monserrat and C.J. Moreno-Ávila.

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A study of skew lie product involving generalized derivations

Muzibur Rahman Mozumder

Let R be a ring with involution σ. Notation $\nabla[t_1, t_2]$ denotes the skew lie product and defined by $(t_1t_2 - t_2\sigma(t_1))$. The main objective of this paper is to investigate the commutativity of σ-prime rings with involution σ of the second kind equipped with skew lie product involving generalized derivation. Finally, we provide some examples to demonstrate that the conditions assumed in our results are not unnecessary.

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Split absolutely irreducible integer-valued polynomials over discrete valuation domains

Sarah Nakato

Let D be a domain with quotient field K. The ring of integer-valued polynomials on D,

$$\mathrm{Int}(D) = \{f \in K[X] \mid f(D) \subseteq D\},$$

in general is far from having unique factorization of elements into irreducibles. In this talk, we focus on the absolutely irreducible elements of $\mathrm{Int}(D)$ when D is a discrete valuation domain. Recall that an irreducible element of a commutative ring is called absolutely irreducible if none of its powers has more than one (essentially different) factorizations into irreducibles. The concept of absolute irreducibility has been used in several contexts, for instance, in characterizing number fields with certain class groups. In the context of non-unique factorizations into irreducibles, non-absolutely irreducible elements are crucial in investigating patterns of factorizations. In this talk, we discuss a characterization of the absolutely irreducible elements of $\mathrm{Int}(D)$ that split over K.

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On the class semigroup of a class of C-monoids

Jun Seok Oh

A C-monoid is a suitably defined submonoid of a factorial monoid with finite class semigroup. This monoid plays a key role in an arithmetical investigation of a large class of Mori domains. It is well understood that a C-monoid is Krull if and only if the class semigroup coincides with the (v-)class group of a Krull monoid, and the arithmetic of a Krull monoid can be determined by the structure of its (v-)class group. The finiteness of the class semigroup allows us to prove the similar arithmetical finiteness for a general C-monoid to results known in the Krull case. Recently, the algebraic structure of the class semigroup has begun to be studied for a non-Krull C-monoid. In this talk, we study the structure of the class semigroup of the specific (non-Krull) C-monoids.

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Affine Semigroups of Maximal Projective Dimension

Om Prakash

A submonoid of $\mathbb{N}^d$ is of maximal projective dimension (MPD) if the associated semigroup k-algebra has the maximum possible projective dimension. Such submonoids have a non-trivial set of pseudo-Frobenius elements. We generalize the notion of symmetric numerical semigroups, and pseudo-symmetric numerical semigroups to the case of MPD-semigroups in $\mathbb{N}^d$, and give their characterizations. Under suitable conditions, we prove that these semigroups satisfy the Extended Wilf's conjecture. We also give a class of MPD semigroups in $\mathbb{N}^d$, where the cardinality of the set of pseudo-Frobenius elements may not be a bounded function of its embedding dimension.

This is a joint work with Kriti Goel and Indranath Sengupta.

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Noetherian-like properties and zero-dimensionality in some extensions of rings

Omar Ouzzaouit

This is a joint work with Hwankoo Kim and Ali Tamoussit. We investigate the transfer of the locally Noetherian property to flat overrings, Serre and Nagata rings, trivial ring extensions, the finite direct product of rings, and amalgamated duplications. We also examine the transfer of the Q-Noetherian property to the Nagata ring and amalgamated duplications, and we provide an analog to the Eakin-Nagata Theorem. We define the locally Q-Noetherian property and study its transfer to flat overrings, the finite direct product of rings, the Nagata ring, trivial ring extensions, and amalgamated duplications. We also introduce the notion of the Q-Artinian ring, and we prove that this class of rings coincides with 0-dimensional rings. Particular attention is devoted to the study of Q-strongly 0-dimensional rings.

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Krull property of generalized power series rings

Mi Hee Park

Let D be an integral domain and let $(S, \leq)$ be a torsion-free, $\leq$-cancellative, subtotally ordered monoid. We show that the generalized power series ring $[\![D^{S,\leq}]\!]$ is a Krull domain if and only if D is a Krull domain and S is a Krull monoid.

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Polynomial Dedekind Domains

Giulio Peruginelli

We are interested in characterizing Polynomial Dedekind domains over the ring of integers, that is, Dedekind domains R between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$. For a prime $p \in \mathbb{Z}$, let $\overline{\mathbb{Z}_p}$ be the absolute integral closure of the ring of p-adic integers $\mathbb{Z}_p$. If we suppose that the residue fields of prime characteristic of R are finite fields, then we show that, for each prime $p \in \mathbb{Z}$, there exists a finite subset E_p of $\overline{\mathbb{Z}_p}$ of transcendental elements over $\mathbb{Q}$ such that R is formed by polynomials which are simultaneously integer-valued over E_p for each prime p, that is, $R = \{f \in \mathbb{Q}[X] \mid f(E_p) \subseteq \overline{\mathbb{Z}_p}, \forall \text{ prime } p \in \mathbb{Z}\}$.
As a side result, we also characterize the PIDs with finite residue fields of prime characteristic contained between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$.
Finally, we show that given a group G which is the direct sum of a countable family of finitely generated abelian groups, there exists a Polynomial Dedekind domain R, $\mathbb{Z}[X] \subset R \subseteq \mathbb{Q}[X]$, such that the class group of R is G.

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The $D + M$ construction in semidomains

Harold Polo

The $D + M$ construction (introduced by Gilmer in the context of valuation domains and further studied by Brewer and Rutter in the more general context of integral domains) is an important source of examples in commutative ring theory. We study a generalization of this construction for *semidomains* (i.e., subsemirings of integral domains): Let $T = K+M$ be a semidomain, where K is a yoked semifield and M is a maximal ideal of T; for a subsemidomain D of K, set $S = D + M$. This is joint work with Felix Gotti.

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Bhargava factorials and irreducibility of integer-valued polynomials

Devendra Prasad

The ring of integer-valued polynomials over a given subset S of $\mathbb{Z}$ (or $\mathrm{Int}(S, \mathbb{Z})$) is defined as the set of polynomials in $\mathbb{Q}[x]$ which maps S to $\mathbb{Z}$. In this talk, we discuss how Bhargava factorials can be used to check the irreducibility of a given polynomial $f \in \mathrm{Int}(S, \mathbb{Z})$.

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A characterization of half-factorial orders in algebraic number fields

Balint Rago

We give an algebraic characterization of half-factorial orders in algebraic number fields. This generalizes prior results for seminormal orders and for orders in quadratic number fields.

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On some combinatorial invariants associated with commutative rings

Rameez Raja

Let $G = (V, E)$ be a simple graph realized by a commutative ring R with unity. We discuss association of a combinatorial invariant (young diagram) with R. We study Laplacian of a combinatorial structure G and show that the representatives of some algebraic invariants are eigenvalues of the Laplacian of G. Let $m, n \in \mathbb{Z}_{>0}$ be two positive integers. The Young's partition lattice $L(m, n)$ is defined to be the poset of integer partitions $\mu = (0 \leq \mu_1 \leq \mu_2 \leq \cdots \leq \mu_m \leq n)$. We can visualize the elements of this poset as Young diagrams ordered by inclusion. We conclude with a discussion on Stanley's conjecture regarding symmetric saturated chain decompositions (SSCD) of $L(m, n)$.

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Associated primes of powers of monomial ideals

Jutta Rath

To every ideal I in a ring one can associate a unique set of prime ideals, the so-called associated primes of I. In many settings, these primes can be interpreted as structure revealing building blocks of I. The associated primes of an ideal in $\mathbb{Z}$ correspond to the prime divisors of the generator of the ideal; the associated primes of the inducing ideal of an algebraic variety correspond to the its irreducible components; for the edge ideal of a finite simple graph, the associated primes are precisely the prime ideals generated by the minimal vertex covers of the graph. Furthermore, also associated primes of *powers* of ideals have a connection to graph theory: The index after which the sequence of associated primes of powers of the cover ideal of a graph is constant gives an upper bound for the chromatic number of the underlying graph. The phenomenon of occurring changes in the set of associated primes when considering powers of an ideal has been observed in many different settings. In the Noetherian case, it is well known that the sequence of associated primes of powers of an ideal stabilizes. This talk focuses on the stabilization of associated primes of powers of monomial ideals. A technique to develop upper bounds for the power of an ideal after which the sequence is non-increasing is presented.

The results presented in this talk are joint work with R. Rissner and C. Heuberger.

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Noncommutative tensor triangulated categories and coherent frames

Samarpita Ray

In this talk, I will discuss a point-free approach for constructing the Nakano-Vashaw-Yakimov-Balmer spectrum [3] of a noncommutative tensor triangulated category under some mild assumptions. This approach is built on ideas inspired from the work of Koch and Pitsch [1] and methods from non-commutative ring theory. In particular, I will provide a conceptual way of classifying radical thick tensor ideals of a noncommutative tensor triangulated category using frame theoretic methods, recovering the universal support data in the process. This is a recent joint work [2] with Vivek Mohan Mallick.

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On the zero-sum invariants over $C_n \rtimes_s C_2$

Sávio Ribas

Let $n \in \mathbb{N}_{\geq 8}$ which is neither an odd prime power nor twice an odd prime power. Let $s \in \mathbb{Z}$ such that $s^2 \equiv 1 \pmod{n}$ and $s \not\equiv \pm 1 \pmod{n}$. Set

$$G = C_n \rtimes_s C_2 = \langle x, y \mid x^2 = y^n = 1, yx = xy^s \rangle,$$

a non-abelian metacyclic group that is distinct from the dihedral group. In some recent works (see [2, 3]), we determined, for almost all possible pairs (n, s), the direct and the inverse zero-sum problems over G, namely

- the small Davenport constant $\mathsf{d}(G)$, defined as the maximal length of all product-one free sequences over G,
- the eta constant $\eta(G)$, defined as the smallest length such that every sequence over G of such length has a product-one subsequence of length at most $\exp(G)$,
- the Gao constant $\mathsf{E}(G)$, defined as the smallest length such that every sequence over G of such length has a product-one subsequence of length $|G|$, and
- the Erdős-Ginzburg-Ziv constant $\mathsf{s}(G)$, defined as the smallest length such that every sequence over G of such length has a product-one subsequence of length $\exp(G)$.

In this talk, we will expose the main ideas of the proofs, which are based on the inductive method. We will further propose some problems.
This is a joint work with D.V. Avelar and F.E. Brochero Martínez.

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Powers of irreducibles in rings of integer-valued polynomials

Roswitha Rissner

Non-unique factorization of elements into irreducibles has been observed in the ring of integer-valued polynomials and its generalizations. It is known that every (multi-)set consisting of integers greater than 1 can be realized as the (multi-)set of lengths of an integer-valued polynomial over a Dedekind domain D with infinitely many maximal ideals whose residue fields are finite. Moreover, under the same assumptions on D, $\mathrm{Int}(D)$ is not transfer Krull. The proofs of these two statements are build on the constructions of integer-valued polynomial whose factorization behaviour can be fully controlled. For this, it is crucial to avoid the situation of a factorization in which an irreducible factor occurs more than once. This is because in non-unique factorization domains there is in general no saying how the powers of an irreducible element factor. From a factorization-theoretic point of view, one wants therefore identify those elements among the irreducibles whose powers factor uniquely. We call such elements *absolutely irreducible.* This talk provides an overview on recent results characterizing absolutely irreducible elements in rings of integer-valued polynomials.

The results presented in this talk are joint work with S. Frisch, M. Hiebler, S. Nakato, and D. Windisch.

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On polynomial extensions of Zaks domains

Moshe Roitman

A Zaks domain is an atomic domain R that is contained in a factorial domain D such that each irreducible element of R remains irreducible in D. Thus, a Zaks domain is half-factorial. We characterize the Zaks property of polynomial extensions. We present a non-factorial Mori domain, such that all its polynomial extensions are Zaks domains. However, a Krull domain or a noetherian domain having a Zaks proper polynomial extension is factorial.

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On the ideal categories of a Noetherian ring

Paracackal G. Romeo

Here we describe the categories $\mathbb{L}_R$, $[\mathbb{R}_R]$ of left [right] principal ideals of a Noetherian ring R with unity. Then the category $\mathbb{L}_R$ has object set $v\mathbb{L}_R$ the principal left ideals generated by idempotents and morphisms appropriate R-linear transformations and similarly the category $\mathbb{R}_R$. It is shown that these are preadditive categories with zero object and are full subcategories of the R-modulue category. Further we investigate the properties of these categories and it is seen that these are categories with subobjects and the morphisms admits factorization property.

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On the number of relations of a numerical semigroup

Alessio Sammartano

How many relations can a numerical semigroup $S \subseteq \mathbb{N}$ have? The answer to this apparently simple question is surprisingly complicated, and our knowledge is very limited. In this talk, I will discuss the problem of finding upper bounds for the number of relations of a numerical semigroup in terms of various invariants, such as the multiplicity, embedding dimension, and width.
This talk is based on joint work with Giulio Caviglia and Alessio Moscariello.

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Separating Noether number of abelian groups

Barna Schefler

The *Noether number* of a finite group is the general upper bound for the degrees of the generators of the algebras of polynomial invariants associated with the finite dimensional representations of the group. For an abelian group it coincides with the *Davenport constant*; this observation highlights the relation between invariant theory and the theory of zero-sum sequences. In recent years the study of separating systems of polynomial invariants (as a relaxation of the concept of generating systems) received much attention. In particular, the notion of the *separating Noether number* of a finite group was introduced. In the talk we shall focus on the case of abelian groups and discuss results concerning questions on zero-sum sequences that have relevance for the separating Noether number of finite abelian groups.

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A generalized notion of cross number and applications to monoids of weighted zero-sum sequences

Wolfgang A. Schmid

Let $(G, +)$ be a finite abelian group. For a sequence $S = g_1 \dots g_l$ over G the cross number of S, denote by $\mathsf{k}(S)$, is defined as $\mathsf{k}(S) = \frac{1}{\mathrm{ord}(g_1)} + \cdots + \frac{1}{\mathrm{ord}(g_1)}$. The concept of cross number is a key-tool in investigations of monoids of zero-sum sequences over finite abelian groups. (The definition extends to abelian torsion groups and more generally to sequences of elements of finite order in not necessarily commutative groups.)
More recently, the monoids of weighted zero-sum sequences have been studied. Let Ω we a set of weights, that is a non-empty set of integers, or more generally of endomorphisms of G. One says that a sequence $S = g_1 \dots g_l$ over G is an Ω-weighted zero-sum sequence if there exist $\omega_i \in \Omega$ such that $\omega_1 g_1 + \cdots + \omega_l g_l = 0$. The set of Ω-weighted zero-sum sequences over G is a submonoid of the monoid of all sequences over G and one is interested in understanding its arithmetic. This problem was recently considered in [1] and [2].
While it is possible to consider the cross numbers of such sequences, it turns out that the traditional concept of cross number is not really well-suited to obtain results in the context of weighted zero-sum sequences. We propose a generalized definition of cross number and present some first applications in the context of weighted zero-sum sequences.
This is joint work with K. Merito and O. Ordaz.

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Leavitt Path Algebras and Talented Monoids via Lie Brackets and Adjacency Matrices

Alfilgen Sebandal

Given a directed graph, one can associate two algebraic entities: the Leavitt path algebra and the talented monoids. The Graded Classification Conjecture of Leavitt path algebras claims that classification of talented monoids is equivalent to graded equivalence of categories of graded modules over the Leavitt path algebra. Hence, correspondence of algebraic structures would then be evidences of the conjecture. In this talk, we shall take a look at these correspondence of structures starting with the Lie bracket algebra of the Leavitt path algebra in relation to the talented monoids. In particular, we characterize balloons in a graph in terms of talented monoids. This then provide a direct link to the Lie simplicity of Leavitt path algebra. Furthermore, we shall see an adjacency-matrix representation of the talented monoids. In particular, we characterize hereditary saturated sets. Since such sets generate the so-called $\mathbb{Z}$-order ideals of the talented monoid and the graded ideals of the Leavitt path algebra, such characterization in terms of matrices could provide a practical tool in studying the structures of the two algebraic entities. Furthermore, we shall see that basis elements of the Leavitt path algebra can be computed using the adjacency matrix. This is then found useful for concepts that look at certain lengths of the products in the algebra such as entropy. Finally, we shall see a confirmation of the conjecture in the finite-dimensional case.

The contents of this talk is a compilation of joint works with Roozbeh Hazrat, Wolfgang Bock, and Jocelyn Vilela.

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Derived set-like constructions in commutative algebra

Dario Spirito

The derived set of a topological space X is the set of all the points of X that are not isolated; this construction can be iterated, obtaining a chain of closed subsets of X indexed by the ordinal numbers. In this talk, I will present two algebraic analogues of this construction.

The first one is constructed from *pre-Jaffard families*, a generalization of the concept of Jaffard families. As a first example, we show how the derived-set like construction allows to generalize the classification of stable semistar operations and singular length functions from Jaffard to pre-Jaffard families. Subsequently, we consider the Picard group of the ring $\mathrm{Int}(D)$ of integer-valued polynomials on a domain D: we show that the results about the localization of $\mathrm{Pic}(\mathrm{Int}(D))$ when D is a Dedekind domain can be generalized to Jaffard families of arbitrary domains, and that under some additional hypothesis they can be further expanded to cover the pre-Jaffard family case.

The second analogue involves almost Dedekind domains: starting from the concept of SP-domains and critical maximal ideals, we show how it is possible to study the group $\mathrm{Inv}(D)$ of the invertible ideals of D through a derived set-like construction, expressing it as a direct sum of groups of continuous functions; in particular, this shows that $\mathrm{Inv}(D)$ is free for every almost Dedekind domain.

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On non-associative algebras generated by gyrogroups

Teerapong Suksumran

In this talk, we mention a non-associative group-like structure, called a *gyro-group*. Then we give a construction of a unital non-associative algebra induced by a finite gyrogroup, called a *gyrogroup algebra*, which is a generalization of a group ring. We also study some algebraic properties of gyrogroup algebras and construct a module over a gyrogroup algebra.

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Essential properties of Integer-valued polynomial rings

Francesca Tartarone

If D is a domain with quotient field K, then $\mathrm{Int}(D) := \{f(X) \in K[X] \mid f(D) \subseteq D\}$ is the ring of integer-valued polynomial rings over D.
A globalization of the concept of valuation domain is obtained through the definition of Prüfer domain (H. Prüfer, 1932), i.e. a domain whose localizations at prime ideals are valuation domains. Prüfer v-multiplication domains further generalizes the Prüfer domains using the tool of t-operation on ideals. This generalization allows to involve in this class of domains the polynomial rings $D[X]$, which are never Prüfer (unless D is a field).

Given a subset $\mathcal{P}$ of $\mathrm{Spec}(D)$, D is an *essential domain with defining family* $\mathcal{P}$ if $D = \bigcap_{\mathfrak{p} \in \mathcal{P}} D_{\mathfrak{p}}$ and each $D_{\mathfrak{p}}$ is a valuation domain. Prüfer domains and PvMD's are example of essential domains.

In [1] J. Mott and M. Zafrullah introduced the notion of *P-domain* as an integral domain D such that $D_{\mathfrak{p}}$ is a valuation domain for each associated prime ideal $\mathfrak{p}$ of D. In particular, a P-domain D is essential with defining family $\mathrm{Ass}(D)$. The authors also showed that P-domains are exactly the integral domains such that their rings of fractions are essential domains (equivalently, $D_{\mathfrak{p}}$ is essential for all prime ideals $\mathfrak{p}$ of D). Thus, these domains are also called *locally essential domains*.

In collaboration with Ali Tamoussit, I characterized the integer-valued polynomial rings that are P-domains and we also gave sufficient conditions for $\mathrm{Int}(D)$ to be essential.

I will go through these results extending the talk to an overview of the essential-like properties of $\mathrm{Int}(D)$.

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The Hausdorff spectrum of p-adic analytic pro-p groups

Anitha Thillaisundaram

The concept of Hausdorff dimension first arose in the context of fractals. With time it was also applied to other areas of mathematics, and this was extended in the 90s to profinite groups. A special class of profinite groups includes the compact R-analytic groups, where R is a pro-p domain for a prime p. Recall that a pro-p domain is a local Noetherian integral domain which is complete with respect to the metric defined by the maximal ideal and whose residue field is finite of characteristic p. The R-analytic groups have a rich geometric and analytic structure. Setting $R = \mathbb{Z}_p$ gives the more well-studied p-adic analytic groups, which play a key role in the theory of groups. It is an open question still whether p-adic analytic groups can be characterised by the set of Hausdorff dimensions of their subgroups, i.e. the so-called Hausdorff spectrum. In this talk, we survey what has been done towards answering this question, and we include recent results concerning the Hausdorff spectrum when computed with respect to the lower p-series, the proof of which uses the perspective of Lie algebras and modules. This is joint work with Iker de las Heras and Benjamin Klopsch.

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Atomicity and Subatomicity in Rank-2 Lattice Monoids

Marcos Manuel Tirador

A lattice monoid is a submonoid of a finite-rank free abelian group. Unlike affine monoids, lattice monoids may not be atomic or finitely generated. Atomicity in rank-2 lattice monoids has been studied recently by various authors, including F. Gotti and G. Lettl. In this talk, we will discuss some atomic aspects of rank-2 lattice monoids, including a sufficient condition for atomicity. In addition, we will discuss some properties weaker than being atomic in the same context of rank-2 lattice monoids. This talk is based on a recent paper co-authored with Caroline Liu and Pedro Rodgriguez.

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Power monoids and a conjecture of Bienvenu and Geroldinger

Salvatore Tringali

Endowed with the binary operation of setwise multiplication induced by M, the family of all finite subsets of a (multiplicatively written) monoid M containing the identity 1_M is itself a monoid, herein called the reduced power monoid of M.

Let H and K be monoids and assume K is positively orderable, meaning that there is a total order $\preceq$ on K such that (i) $1_K \preceq x$ for every $x \in K$ and (ii) $x \prec y$ implies $uxv \prec uyv$ for all $u, v \in K$. We show that the reduced power monoid of H is isomorphic to the one of K (if and) only if H is isomorphic to K. In the special case when H and K are numerical monoids, this proves (and extends) a conjecture of P.-Y. Bienvenu and A. Geroldinger.

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Graphs of the Jacobson Radical

Gulsen Ulucak

In this paper, we introduce the unit-Jacobson graph and the idempotent-Jacobson graph. The first graph is defined by the unit elements and the elements of the Jacobson radical of a commutative ring R with nonzero identity and the other graph by the idempotent elements and the elements of the Jacobson radical of R. We give relationships between these new graph concept and some special rings, such as j-clean rings, UJ-rings, local rings, and cartesian rings. Moreover, we investigate the concepts of the dominating set, diameter, and girth on the unit-Jacobson graph.

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V-domain constructions using a Kolchin type universal field

John van den Berg

This is joint work with Pham Ngoc Anh. All known examples of V-domains are one of three types (or localizations thereof): (1) $D \otimes_K K(x)$ where D is a skew field with centre K, (2) the ring $\mathcal{D}_K$ of differential polynomials over differential field K, and (3) the twisted Laurent polynomial ring $K[x, x^{-1}, \sigma]$ where σ is an automorphism on field K. But in each of these cases, stringent conditions need to be imposed on the ground field K in order that the corresponding overring be a V-domain.

In the case of (1), as Resco [1987] has shown, D must possess the so-called 'Conjugacy Property' over K.

For (2), the differential field K must be 'universal' in the sense that it admits a solution to every linear differential equation, an observation made by Cozzens [1970]. Although motivated by quite different needs, such universal fields had already been constructed much earlier by Kolchin in his pioneering work in differential algebra. These were put to use by Cozzens to produce the first examples of V-domains.

In the case of (3), the analogue of a Kolchin universal field is a field K equipped with automorphism σ such that every linear *sigma-equation* of the form

$$b + a_0 t + a_1\sigma(t) + \cdots + a_n\sigma^n(t) = 0$$

possesses a solution for t in K. Osofsky [1971] noted that if K is a field of prime characteristic and σ the associated Frobenius map, then the above sigma-equation is an instance of an algebraic equation over K, which admits solutions if K is algebraically closed. Osofsky then showed that the field K could be chosen in such a manner that homogeneous equations of the above type admit no nonzero solutions. This restricted form of algebraic closedness produced a twisted Laurent polynomial ring and V-domain having distinct simples, a property not possessed by Cozzens' example.

In this paper a systematic method is developed in the spirit of Kolchin's construction of the universal field, which shows that each field K equipped with a fixed automorphism σ may be extended to a field $\widehat{K}$ and automorphism $\widehat{\sigma}$ with the property that every sigma-equation in $\widehat{K}$ with respect to $\widehat{\sigma}$, admits a solution in $\widehat{K}$. The consequence is a rich supply of new examples of V-domains of type (3).

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Determinantal zeros and factorization of noncommutative polynomials

Jurij Volčič

Hilbert's Nullstellensatz about zero sets of polynomials is one of the most fundamental correspondences between algebra and geometry. More recently, there has been an emerging interest in polynomial equations and inequalities in several matrix variables, prompted by developments in control systems, quantum information theory, operator algebras and optimization. The arising problems call for a suitable version of (real) algebraic geometry in noncommuting variables; with this in mind, the talk addresses matricial sets where noncommutative polynomials attain singular values, and their algebraic counterparts.
Given a noncommutative polynomial f in d noncommuting variables, its free (singularity) locus $\mathcal{Z}(f)$ is the set of all d-tuples of square matrices X such that $f(X)$ is singular. One should think of $\mathcal{Z}(f)$ as a noncommutative analog of a hypersurface in algebraic geometry. The talk explains the interplay between geometry of free loci (irreducible components, inclusion relations, eigenlevel sets, smooth points) and factorization in the free algebra on d symbols. In particular, a Nullstellensatz for free loci is presented, as well as a noncommutative variant of Bertini's irreducibility theorem. Applications and open problems are also discussed along the way.

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Null ideals of subsets of matrix rings

Nicholas Werner

For a ring R and $S \subseteq R$, the null ideal of S in R is $N_R(S) := \{f \in R[x] \mid f(s) = 0 \text{ for all } s \in S\}$. When R is commutative, it is trivial to verify that S is an ideal of $R[x]$. If R is noncommutative, then one may define $N_R(S)$ as above. However, in this case, $N_R(S)$ may fail to be a two-sided ideal of $R[x]$. In this talk, we will discuss conditions on R and S that guarantee that $N_R(S)$ is two-sided ideal of $R[x]$, and mention a connection to rings of integer-valued polynomials. Particular attention will be paid to the case where R is a ring of $n \times n$ matrices with entries from a field.

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Monoids of Modules, 1

Roger Wiegand

We study direct-sum decompositions of modules over a Noetherian ring R using the monoid add M, which consists of isomorphism classes $[X]$ of R-modules X that are isomorphic to direct-summands of the direct sum of some finite direct sum of copies of M. We give examples of weird decompositions, and also state some rules that are always satisfied. Also, there are examples to show that many desired rules can fail miserably. This is part I, to be continued by Sylvia Wiegand in part 2.

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Monoids of Modules, 2

Sylvia Wiegand

We study direct-sum decompositions of modules over a Noetherian ring R using the monoid add M, which consists of isomorphism classes $[X]$ of R-modules X that are isomorphic to direct-summands of the direct sum of some finite direct sum of copies of M. We give examples of weird decompositions, and also state some rules that are always satisfied. Also, there are examples to show that many desired rules can fail miserably. This is part 2, a continuation of part 1, given by Roger Wiegand.

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Geometry of schemes and arithmetic of their associated rings

Daniel Windisch

In this talk, I will try to emphasize a possible connection of geometric properties of (integral) schemes and the behaviour of multiplicative factorizations of the elements of their local and global rings as products of irreducible elements. The linking piece between the two supposedly unrelated topics is the theory of divisors that is important in both.

Samuel conjectured in 1961 that a (Noetherian) local complete intersection ring which is a UFD (unique factorization domain) in codimension at most three is itself a UFD. It is said that Grothendieck invented local cohomology to prove this fact. Following the philosophy that a UFD is nothing else than a Krull domain with trivial divisor class group, I will talk about the following generalization:
Suppose that A is a normal domain and a complete intersection. Then the divisor class group of A is a subgroup of the projective limit of the divisor class groups of the localizations A_p, where p runs through all prime ideals of height at most 3 in A.

Implications of this result on the relation of Weil and Cartier divisors of integral separated Noetherian schemes are presented.
Moreover, based on a seemingly forgotten theorem of Samuel from the 60s, I will propose a connection (again via divisors) between the existence of rational points on an algebraic variety and non-unique factorizations of its coordinate ring.

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On subalgebras of matrix algebra satisfying some polynomial identity

Michal Ziembowski

A subalgebra of the full matrix algebra $\mathrm{M}_n(K)$, K a field, satisfying the identity

$$[x_1, y_1]\,[x_2, y_2] \cdots [x_q, y_q] = 0$$

is called a D_q subalgebra of $\mathrm{M}_n(K)$.
We describe explicitly, up to conjugation, the structure of maximal D_q subalgebras of $\mathrm{M}_n(K)$ as block triangular subalgebras of $\mathrm{M}_n(K)$ with maximal commutative diagonal blocks. The sizes of the diagonal blocks are shown to play critical in deciding when two maximal D_q subalgebras of $M_n(K)$ are isomorphic. In case K is algebraically closed, we invoke Jacobson's characterization of maximal commutative subalgebras of $\mathrm{M}_n(K)$ with maximum (K-)dimension to obtain a complete classification of maximal D_q subalgebras of $\mathrm{M}_n(K)$ which are conjugates of block triangular subalgebras of $\mathrm{M}_n(K)$ with commutative diagonal blocks of maximum dimension.

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Index