Conference on Rings and Factorizations

Graz, Austria · 19–23 Feb 2018

Programme and Abstracts

E-mail: rings2018@uni-graz.at
Web: http://imsc.uni-graz.at/rings2018/

UNI GRAZ · NAWI Graz · TU Graz · Doctoral Program Discrete Mathematics, TU & KFU Graz · MU Leoben, Austria · FWF Der Wissenschaftsfonds. · Das Land Steiermark

Welcome

It is our great pleasure to welcome you to

CRF2018: A Conference on Rings and Factorization.

The conference will take place from February 19th to 23rd, 2018 at:

Institute of Mathematics and Scientific Computing
Karl-Franzens-Universität Graz (UniGraz)
Heinrichstraße 36, 8010 Graz, Austria

We are grateful to the following sponsors for their financial support:

- University of Graz.
- NAWI Graz.
- Graz University of Technology.
- Doctoral Program in Discrete Mathematics.
- Austrian Science Fund (FWF).
- The State of Styria.

We thank Florian Kainrath for his help with the booklet, Daniel Smertnig for creating and maintaining the conference website, and our secretary, Andrea Sartori, for her help with the administrative part of the organization.

This booklet has been put together to help you find your way through the conference and make the most of your stay in Graz. Additional information can be found online at the URL:

http://imsc.uni-graz.at/rings2018/

During the week, we can be recognized by the green background of our badge. Please come and talk to us if you have any questions.

The local organizers

Alfred Geroldinger · Jun Seok Oh · Salvatore Tringali · Qinghai Zhong

Plenary Speakers

- Phạm Ngọc Ánh (Alfréd Rényi Institute of Mathematics, Hungary)
- Carmelo Finocchiaro (Università di Padova, Italy),
- David J. Grynkiewicz (University of Memphis, USA),
- Evan Houston (UNC Charlotte, USA),
- Mi Hee Park (Chung-Ang University, Korea),
- Pavel Příhoda (Charles University, Czech Republic),
- Manuel Reyes (Bowdoin College, USA),
- Luigi Salce (Università di Padova, Italy),
- Daniel Smertnig (University of Waterloo, Canada).

Scientific Advisory Board

- Gyu Whan Chang (University of Incheon, Korea)
- Alberto Facchini (Università di Padova, Italy)
- Marco Fontana (Università degli Studi "Roma Tre", Italy)
- Sophie Frisch (Graz University of Technology, Austria)
- K. Alan Loper (The Ohio State University, USA)

List of participants

Adnan Abbasi (Aligarh Muslim University, India)
Sofia Afanaseva (St. Petersburg State University, Russia)
Amr Ali Abdulkader Al-Maktry (Graz University of Technology, Austria)
Meltem Altun-Ozarslan (Università di Padova, Italy)
Gerhard Angermüller (Germany)
Phạm Ngọc Ánh (Alfréd Rényi Institute of Mathematics, Hungary)
Austin Antoniou (The Ohio State University, USA)
Maria José Arroyo-Paniagua (Universidad Autonoma Metropolitana, Mexico)
Ayman Badawi (American University of Sharjah, United Arab Emirates)
Paul Baginski (Fairfield University, USA)
Mashhour Bani-Ata (PAAET-Kuwait, Kuwait)
Valentina Barucci (Sapienza Università di Roma, Italy)
Charlie Beil (University of Graz, Austria)
Paul-Jean Cahen (Aix-Marseille Université, France)
Federico Campanini (Università di Padova, Italy)
Jose Capco (Johannes Kepler University Linz, Austria)
Jean-Luc Chabert (University of Picardie, France)
Gyu Whan Chang (Incheon National University, Republic of Korea)
Nega Chere (Addis Ababa University, Ethiopia)
Andrada Cîmpean (Babeş-Bolyai University, Romania)
Cory H. Colbert (Williams College, USA)
Laura Cossu (Università di Padova, Italy)
Kalman Cziszter (Alfréd Rényi Institute of Mathematics, Hungary)
Marco D'Anna (Università di Catania, Italy)
Mátyás Domokos (Alfréd Rényi Institute of Mathematics, Hungary)
Tiberiu Dumitrescu (University of Bucharest, Romania)
Salma Elaoud (University of Science at Tunis, Tunisia)
Sabine Evrard (University of Picardie, France)
Alberto Facchini (Università di Padova, Italy)
Yushuang Fan (China University of Geosciences, Beijing, China)
Carmelo Antonio Finocchiaro (Università di Padova, Italy)
Marco Fontana (Università degli Studi "Roma Tre", Italy)
Sophie Frisch (Graz University of Technology, Austria)
Stefania Gabelli (Università degli Studi "Roma Tre", Italy)
Ana Garcia Elsener (University of Graz, Austria)
Pedro A. García-Sánchez (Universidad de Granada, Spain)
Alfred Geroldinger (University of Graz, Austria)
Felix Gotti (University of California, Berkeley, USA)
Badreddine Gouaid (University of Gafsa, Tunisia)
David Grynkiewicz (University of Memphis, USA)
Lorenzo Guerrieri (Università di Catania, Italy)
Ashok Ji Gupta (Indian Institute of Technology BHU, India)
Ashish Gupta (Vivekananda University, India)
Franz Halter-Koch (University of Graz, Austria)

Ahmed Hamed (University of Monastir, Tunisia)
Amor Haouaoui (University of Monastir, Tunisia)
Richard Erwin Hasenauer (Northeastern State University, USA)
Dolors Herbera (Universitat Autònoma de Barcelona, Spain)
Evan Houston (University of North Carolina at Charlotte, USA)
Ali Jaballah (University of Sharjah, United Arab Emirates)
Hwankoo Kim (Hoseo University, Republic of Korea)
Handan Kose (Ahi Evran University, Turkey)
Dijana Kreso (University of British Columbia, Canada)
Paolo Leonetti (Università Bocconi, Italy)
Yuanlin Li (Brock University, Canada)
David Llena (Universidad de Almeria, Spain)
Alan Loper (Ohio State University, USA)
Thomas Lucas (Univeristy of North Carolina at Charlotte, USA)
Ahmed Maatallah (University of Monastir, Tunisia)
Ali Majidinya (Salman Farsi University of Kazeroun, Iran)
Tusif Ahmed Malik (GC University Lahore, Pakistan)
László Márki (Alfréd Rényi Institute of Mathematics, Hungary)
Eshita Mazumdar (Nankai University, China)
Muzammil Mukhtar (GC University Lahore, Pakistan)
Sarah Nakato (Graz University of Technology, Austria)
Ilia Nekrasov (St. Petersburg State University, Russia)
Jun Seok Oh (University of Graz, Austria)
Abdulhakeem Olayiwola (Sule Lamido University, Nigeria)
Reyes M. Ortiz-Albino (University of Puerto Rico, Puerto Rico)
Omar Ouzzaouit (Cadi Ayyad University, Morocco)
Carlo Pagano (University of Leiden, Netherlands)
Romeo Parackal (CUSAT, India)
Mi Hee Park (Chung-Ang University, Republic of Korea)
Sajad Ahmad Pary (Aligarh Muslim University, India)
Manoj Kumar Patel (National Institute of Technology at Nagaland, India)
Shariefuddin Pirzada (University of Kashmir, India)
Flaviu Pop (Babeş-Bolyai University, Romania)
Pavel Příhoda (Charles University, Czech Republic)
Anam Rani (University of Central Punjab, Pakistan)
Shafiq Ur Rehman (CIIT Attock, Pakistan)
Andreas Reinhart (New Mexico State University, USA)
Manuel Reyes (Bowdoin College, USA)
Sávio Ribas (Instituto Federal de Minas Gerais, Brazil)
Roswitha Rissner (Graz University of Technology, Austria)
Moshe Roitman (University of Haifa, Israel)
Afshan Sadiq (GC University Lahore, Pakistan)
Luigi Salce (Università di Padova, Italy)
Wolfgang Schmid (Université Paris 8, France)
Konrad Schrempf (Graz University of Technology, Austria)
Sampat Kumar Sharma (India)

Mike Siddoway (Colorado College, USA)
Daniel Smertnig (University of Waterloo, Canada)
Dario Spirito (Università degli Studi "Roma Tre", Italy)
Anjum Tariq (Islamia College Univeristy, Pakistan)
Francesca Tartarone (Università degli Studi "Roma Tre", Italy)
Jonathan Toledo (Cinvestav IPN, Mexico)
Salvatore Tringali (University of Graz, Austria)
Uzi Vishne (Bar Ilan University, Israel)
Sergei Vostokov (St. Petersburg State University, Russia)
Bilal Ahmad Wani (Aligarh Muslim University, India)
Roger Wiegand (University of Nebraska, USA)
Sylvia Wiegand (University of Nebraska, USA)
Raza Zahid (University of Sharjah, United Arab Emirates)
Nasr Zeyada (Cairo University, Egypt)
Qinghai Zhong (University of Graz, Austria)

General information

Every lecture room will be equipped with a digital projector, a computer, and blackboards, and one of the organizers will be there on hand to help. Speakers can bring their own laptop.

For general inquiries or technical problems (with projectors, Internet, etc.), please see the registration desk or contact one of the organizers.

Library

There is a library at the 3rd floor of the Institute for Mathematics and Scientific Computing (the conference venue). It is open from Monday to Friday, from 9.00 a.m. to 1.00 p.m.

Internet access

Internet services are available for free. To access the Internet, turn on the Wireless LAN on your computer in the area of the conference premises, and either use your eduroam account, if you have one, or select the network:

"KFU-Tagung"

This network is not secured, so you do not need a password to access it, though depending on your OS you might receive a general security warning for accessing an unsecured network.

Please see the registration desk for an account to use the terminals in the students' lounge on the 2nd floor of the conference venue.

Social Events

On Tuesday, February 20th, 2018 at 7.00 p.m., there will be a reception by the Governor of the Federal State of Styria, in the staterooms of Graz Burg Hofgasse 15, 8010 Graz.

On Thursday, February 22th, 2018 at 5.40 p.m., there will be a Flute & Piano Recital, by Harald Fripertinger (flute) and Svetlana Hübler (piano). The recital will take place in the same room of the plenary talks, at the Institute of Mathematics and Scientific Computing of the University of Graz.

Emergency numbers

In case of an emergency, you may contact one of the following:

- Fire brigade: 122
- Police: 133
- Doctors' emergency service: (+43 316) 141
- Ambulance: 144

For minor emergencies, you may also contact one of the organizers:

- Alfred GEROLDINGER: +43 681 81625821
- Jun Seok OH: +43 681 10834220
- Salvatore TRINGALI: +39 346 3364107
- Qinghai ZHONG: +43 681 81303589

Pharmacies

- Glacis Apotheke: Glacisstraße 31, 8010 Graz, Geidorf (close to the conference venue).
- Apotheke Zu Maria Trost: Mariatroster Straße 31, 8043 Graz, Mariatrost (half-way to Hotel Stoiser, on bus line 58).
- Apotheke zur Göttlichen Vorsehung: Heinrichstraße 3, 8010 Graz, Geidorf (close to the conference venue).

Where to have lunch or dinner

The cafeteria on the ground floor of the conference venue offers simple dishes for lunch, see the next page for details about the menu.

You can also check some daily offers from restaurants in the nearby of the campus through the UniGraz website, at the url

https://menue.uni-graz.at/ (in German)

Besides that, there are restaurants of all levels in the city center. You can learn more about them from

https://www.graztourismus.at/en/eat-and-drink/restaurant-guide

UniCafe Campus · Lunch Menu

Feb 19–23, 2018

Opening hours: Monday - Friday: 8.30 a.m. - 4.00 p.m.
Menu available from 12.00 p.m. on

Mon, Feb 19, 2018

Coconut, ginger & carrot soup (vegetarian)	4,20 €
Pasta with speck & sage	6,20 €
Lasagne with spinach & ricotta	7,20 €
Special dish (hummus, salad, lasagne with spinach & ricotta, and bread)	8,20 €

Tue, Feb 20, 2018

Minestrone (vegetarian)	4,20 €
Pasta with gorgonzola cheese	6,20 €
Baked gnocchi & cheese	7,20 €
Special dish #1 (hummus, salad, lasagne with spinach & ricotta, and bread)	8,20 €
Special dish #2 (chicken tikka masala with rice & lime)	8,20 €

Wed, Feb 21, 2018

Oriental lentil soup (vegetarian)	4,20 €
Pasta with gorgonzola cheese	6,20 €
Sweet potato curry with rice (vegetarian)	7,20 €
Lasagne alla bolognese	7,20 €

Thu, Feb 22, 2016

Red beet & coconut soup (vegetarian)	4,20 €
Pasta with gorgonzola cheese	6,20 €
Fennel & tomato stew with polenta (vegetarian)	7,20 €
Special dish (Italian beef ragù with polenta)	8,20 €

Fri, Feb 23, 2018

Tomato soup (vegetarian)	4,20 €
Pasta with gorgonzola cheese	6,20 €
Chili con carne with potato bread	7,20 €

Contents

Invited talks **1**

Phạm Ngọc Ánh: *An axiomatic divisibility theory for commutative rings* . . . 1

Carmelo Antonio Finocchiaro: *The Smyth powerdomain of a spectral space* . . . 2

David J. Grynkiewicz: *Finiteness of the Elasticities $\rho_k(G_0)$ Inside a Lattice* . . . 3

Evan Houston: *Counting semistar operations on Prüfer domains* . . . 4

Mi Hee Park (NAWI Graz lecture): *Root extension in polynomial and power series rings* . . . 5

Pavel Příhoda: *Monoids of pure projective modules* . . . 6

Manuel Reyes: *Progress with the Prime Ideal Principle* . . . 7

Luigi Salce: *Almost perfect commutative rings* . . . 8

Daniel Smertnig: *Non-unique factorizations in bounded hereditary noetherian prime rings* . . . 9

Contributed talks **11**

Adnan Abbasi: *Commutativity theorems in rings with involutions* . . . 11

Sofia Afanaseva: *Arithmetic of $\mathcal{O}_0$-modules in the case of small ramification* . . . 12

Amr Ali Abdulkader Al-Maktry: *Polynomial functions of the ring of dual numbers modulo m* . . . 13

Gerhard Angermüller: *Roots in extensions of domains* . . . 14

Austin Antoniou: *Especially short sequences with full sumset* . . . 15

Ayman Badawi: *On weakly (m,n)-closed ideals and (m,n)-von Neumann regular rings* . . . 16

Paul Baginski: *Nonunique factorization in the ring of integer-valued polynomials* . . . 17

Mashhour Bani-Ata: *On stabilizers of Tits subspaces of $E_6(K)$ for fields K of characteristic two* . . . 18

Valentina Barucci: *Dilatations of numerical semigroups* . . . 19

Charlie Beil: *Nonnoetherian coordinate rings and their noncommutative resolutions* . . . 20

Paul-Jean Cahen: *Pointwise minimal extensions* . . . 21

Federico Campanini: *On an additive category whose objects have type at most four* . . . 22

Jean-Luc Chabert: *From Pólya fields to Pólya groups* 23
Gyu Whan Chang: *UMT-domain property of graded integral domains* 24
Nega Chere: *The lattice of topologizing filters of rings of continuous functions* 25
Andrada Cîmpean: *Weakly nil-clean index and uniquely weakly nil-clean rings* 26
Laura Cossu: *Products of idempotent matrices over Prüfer domains* 27
Kálmán Cziszter: *New results on the Noether number of finite groups* 28
Marco D'Anna: *Value semigroups and good semigroups* 29
Mátyás Domokos: *On the nilpotency index of nil algebras* 30
Tiberiu Dumitrescu: *Ideal factorization in integral domains* 31
Alberto Facchini: *Factorizations of ideals in noncommutative rings similar to factorizations of ideals in commutative Dedekind domains* 32
Yushuang Fan (NAWI Graz lecture): *Unions of sets of lengths* . . . 33
Ana Garcia Elsener: *Cluster algebras: Factoriality and class groups* . 34
Felix Gotti: *On the sets of lengths of Puiseux monoids* 35
Lorenzo Guerrieri: *Directed unions of local quadratic and monoidal transforms and GCD domains* 36
Ashish Gupta: *Quantum polynomial rings* 37
Ashok Ji Gupta: *A note on direct injective modules* 38
Ahmed Hamed: *On the class group of formal power series rings* . . 39
Richard Erwin Hasenauer: *Factorization in Prüfer domains* 40
Dolors Herbera: *Monoids of countably generated pure-projective modules* 41
Ali Jaballah & Mabrouk Ben Nasr: *From overrings of Prüfer domains to rings in non-integrally closed ring extensions* 42
Hwankoo Kim: *The finitistic weak dimension of a pseudo-valuation domain is at most one* 43
Dijana Kreso: *Polynomial decomposition and linearly recurrent sequences of polynomials* 44
David Llena: *Delta sets of affine semigroups* 45
Yuanlin Li: *On $*$-clean non-commutative group rings* 46
Alan K. Loper: *Ideal theory of integer-valued polynomials in several variables* 47
Tom Lucas: *Valuative Marot Rings* 48
Ali Majidinya: *The Sum-Essential Left Ideal Graph of Rings* 49
Tusif Ahmed Malik: *Radical factorization in rings with zero-divisors* 50
Muzammil Mukhtar: *Commutative rings with two-absorbing factorizations (TAF-rings)* 51
Sarah Nakato: *Non-unique factorizations in rings of integer-valued polynomials on certain Dedekind domains* 52
Abdulhakeem Olayiwola: *Counting distinct fuzzy subgroups of dihedral groups D_{2p^sq}* 53
Reyes Matiel Ortiz-Albino: *A type of generalized factorizations* . . 54

Omar Ouzzaouit: *On Hilbert and Goldman Rings Issued from Amalgamated Algebras* . . . 55
Manoj Kumar Patel: *Fully-invariant Semi-Injective Modules* . . . 56
Shariefuddin Pirzada: *On the metric dimension of a zero-divisor graph associated with a commutative ring* . . . 57
Flaviu Pop: *Cosilting Modules* . . . 58
Anam Rani: *Perinormal rings with zero-divisors* . . . 59
Sharif ur Rehman: *A class of pinched domains with finitely many overrings* . . . 60
Andreas Reinhart: *Radical factorization in ideal systems of monoids* 61
Sávio Ribas: *On extremal product-one free sequences and weighted Davenport constants* . . . 62
Roswitha Rissner: *J-ideals of matrices over PIDs* . . . 63
Moshe Roitman: *On strongly primary monoids and domains* . . . 64
Afshan Sadiq: *Algorithm for primary submodule decomposition without producing intermediate redundant components* . . . 65
Wolfgang A. Schmid: *"Every" set is a set of lengths of a numerical monoid* . . . 66
Konrad Schrempf: *A Factorization Theory for some Free Fields* . . . 67
Dario Spirito: *Jaffard families and extension of star operations* . . . 68
Francesca Tartarone: *Integer-valued polynomial over quaternion algebras* . . . 69
Uzi Vishne: *Decomposition over a division algebra and Ramanujan complexes* . . . 70
Sergey Vostokov & Ilia Nekrasov: *Formal modules in constructible class field theory* . . . 71
Roger Wiegand: *Betti tables over short Gorenstein algebras* . . . 72
Sylvia Wiegand: *Prime ideals in rings of power series and polynomials* 73
Raza Zahid: *On the sandpile group of a subdivision of wheels* . . . 74

Invited talks

An axiomatic divisibility theory for commutative rings

Phạm Ngọc Ánh

In the first part of this talk we discuss the extreme case when ideal theory does not simplify essentially the multiplicative structure of commutative rings. Namely, we present a nice result of Kearnes and Szendrei characterizing rings for which every principal ideal has a unique generator. This lead to the investigation of rings with a group of units having few elements where Mersenne primes appear naturally. In the main part we discuss Bezout monoids as an axiomatic ideal theory of arithmetical rings and we present a structure theory of Bezout monoids established in a join work with Márki and Vámos. Arithmetical rings are invented by Fuchs and become an appropriate class of rings where one can develop a fairly good ideal theory. Bezout monoids are introduced by Boschbach without emphasizing a close relation with arithmetical rings. A satisfactory theory is developed for both semi-hereditary Bezout monoids and Bezout monoids with one minimal prime ideal. The latter lead to a generalization of Clifford's result on naturally totally ordered commutative semigroups. Semi-hereditary Bezout monoids as well as Bezout monoids with one minimal prime filters are representable.

References

[1] K. Kearnes and Á. Szendrei, *private communication*.

[2] P.N. Ánh, *Generalization of a theorem of Clifford*, Acta Math. Vietnam. **41** (2016), no. 3, 471–480.

[3] P.N. Ánh and M. Siddoway, *Divisibility theory of arithmetical rings with one-minimal prime ideal*, Comm. Algebra **44** (2016), no. 2, 823–826.

[4] P. N. Ánh and M. Siddoway, *Divisibility theory of semi-hereditary rings*, Proc. Amer. Math. Soc. **138** (2010), no. 12, 4231–4242.

[5] P.N. Ánh, L. Márki and P. Vámos, *Divisibility theory of commutative rings: Bezout monoids*, Trans. Amer. Math. Soc. **364** (2012), no. 8, 3967–3992.

Alfréd Rényi Institute of Mathematics & Hungarian Academy of Sciences
Reáltanoda utca 13-15, 1053 Budapest, Hungary
E-mail: anh.pham.ngoc@renyi.mta.hu

The Smyth powerdomain of a spectral space

Carmelo Antonio Finocchiaro

Let X be a spectral space and let $\mathcal{X}(X)$ be the collection of all nonempty closed subsets of X, endowed with the inverse topology. In a natural way $\mathcal{X}(X)$ can be seen as a topological space, endowed with the so-called upper Vietoris topology. The aim of this talk, based on a paper in collaboration with M. Fontana and D. Spirito, is to study this topological construction and to provide some applications to Commutative Ring Theory.

Department of Mathematics "Tullio Levi-Civita", Università di Padova
Via Trieste 63, 35131 Padua, Italy
E-mail: carmelo@math.unipd.it

Finiteness of the Elasticities $\rho_k(G_0)$ Inside a Lattice

David J. Grynkiewicz

If H is a Krull monoid with class group G and $G_0 \subseteq G$ is the subset of classes containing prime divisors, then standard theory allows one to study factorial properties of the monoid H via combinatorial aspects of the block monoid $\mathcal{B}(G_0)$, which is the monoid consisting of all zero-sum sequences (finite multi-sets) with terms from G_0. In particular, for an integer $k \geq 1$, we have $\mathcal{U}_k(G_0) := \mathcal{U}_k(\mathcal{B}(G_0)) = \mathcal{U}_k(H)$ and $\rho_k(G_0) := \rho_k(\mathcal{B}(G_0)) = \rho_k(H)$, where $\mathcal{U}_k(H)$ denotes the set of all integers $\ell \geq 1$ for which there are atoms $u_1, \ldots, u_k, v_1, \ldots, v_\ell \in \mathcal{A}(H)$ with $u_1 \cdot \ldots \cdot u_k = v_1 \cdot \ldots \cdot v_\ell$ (thus $\mathcal{U}_k(H)$ contains all factorization lengths obtainable by re-factoring a product of k atoms) and $\rho_k(H) = \sup \mathcal{U}_k(H)$.

When G_0 is finite, many combinatorial and factorial invariants of the monoid H are trivially finite, e.g., $\rho_k(G_0) < \infty$, and likewise when $G_0 = G$ is infinite, these same invariants become trivially infinite. However, it is possible for proper subsets $G_0 \subset G$ to exhibit finite behavior even when G_0 is infinite. Moreover, Claborn's Realization Theorem ensures that essentially any subset $G_0 \subseteq G$ can occur as the subset of classes containing prime ideals for some Dedekind domain H having class group G, and there have been many recent concrete constructions arising from monoids of modules resulting in infinite subsets G_0 of a finitely generated abelian group G. Thus it is of some interest to better understand when, and in what ways, an infinite subset G_0 can behave like a finite one, particularly in regards to the factorial properties of the overlying monoid H.

We focus on the setting when $G_0 \subseteq \Lambda \leq \mathbb{R}^d$ is a subset of a lattice Λ in finite dimensional Euclidean space, so $\Lambda \cong \mathbb{Z}^d$. In this setting, some very basic factorial properties can be translated fully into the language of convex geometry over the field $\mathbb{R}$, though such tentative connections seem to have remained mostly overlooked. Less evident is whether more complicated factorial invariants, such as $\rho_k(G_0)$, could also be fully expressed in the language of Convex Geometry over the field $\mathbb{R}$. We discuss recent work regarding one such affirmative connection in relation to characterizing when $\rho_{d+1}(G_0) < \infty$ is finite, for a subset G_0 of a d-dimensional lattice. Among other consequences, we will see that the finiteness of $\rho_{d+1}(G_0)$ ensures all elasticities $\rho_k(G_0)$ are finite, and that the condition $\rho_{d+1}(G_0) < \infty$ finite is sufficiently general to allow interesting examples of infinite subsets G_0 none-the-less exhibiting finite-like behavior. The majority of the work lies in developing considerable specialized machinery for Convex Geometry regarding asymptotic aspects for the representation of elements as positive linear combinations of elements from a fixed subset G_0 of lattice points.

Department of Mathematical Sciences, University of Memphis
Memphis, TN 38152, USA
E-mail: diambri@hotmail.com, djgrynkw@memphis.edu

Counting semistar operations on Prüfer domains

Evan Houston

A semistar operation on an integral domain R with quotient field K is a closure operation $*$ on the set of nonzero R-submodules of K with the additional property that $(uJ)^* = uJ^*$ for each nonzero element $u \in K$ and each R-submodule J of K. (This differs from the classical star operations in that one does not restrict to fractional ideals nor does one require that $R^* = R$.)

Since their introduction by Okabe and Matsuda in 1994, semistar operations have received a great deal of attention from many authors. In this talk, we consider the problem of "counting" the semistar operations on a Prüfer domain. It is not difficult to show that if R is integrally closed and admits only finitely many semistar operations, then R must be a Prüfer domain. It is also not too difficult to count the number of semistar operations on a valuation domain (Matsuda). For example, if R is a rank-one discrete valuation domain, then R has exactly two semistar operations. However, we use the term "counting" rather loosely: Jesse Elliott has recently shown that the number of semistar operations on a PID with seven maximal ideals exceeds 14 *quintillion*! We survey these and other known results and provide a few new ones (the latter representing joint work with Abdeslam Mimouni and Mi Hee Park).

University of North Carolina at Charlotte
9201 University City Blvd, Charlotte, NC 28223, USA
E-mail: eghousto@uncc.edu

Root extension in polynomial and power series rings

Mi Hee Park

An extension $R \subseteq S$ of commutative rings with unity is called a root extension if for each element $s \in S$, there exists a positive integer n such that $s^n \in R$. Unlike the integral extension, the root extension is not stable under polynomial ring extension. We characterise when the extension $R[X] \subseteq S[X]$ of polynomial rings is a root extension. Using the characterization, we can give a positive answer to the question posed by D. D. Anderson, T. Dumitrescu, and M. Zafrullah in 2004, i.e., $R[X] \subseteq S[X]$ being a root extension implies that $R[X,Y] \subseteq S[X,Y]$ is a root extension. We also characterize when the extension $R[\![X]\!] \subseteq S[\![X]\!]$ of power series rings is a root extension.

Chung-Ang University
84 Heukseok-ro, Dongjak-gu, Seoul, Republic of Korea
E-mail: mhpark@cau.ac.kr

Monoids of pure projective modules

Pavel Příhoda

If R is a ring and $V(R)$ is the set of isoclasses of finitely generated projective right R-modules, then $V(R)$ has a natural structure of a commutative monoid: $[P] + [Q] = [P \oplus Q]$. This monoid is reduced and has an archimedean element $[R]$. It is a well-known result of Bergman that every commutative monoid having these two restrictions is isomorphic to $V(R)$ of some ring. On the other hand $V(R)$ may reflect some properties of R. For example, it is known that if R is semilocal, then $V(R)$ is a Krull monoid, if R is Von Neumann regular, then $V(R)$ is a refinement monoid.

There are other monoids or semigroups one may assign to a ring. For example $V^*(R)$ or $V^*(R) \setminus V(R)$, where $V^*(R)$ is the set of isoclasses of countably generated projectives. Also these monoids may reflect properties of R. E.g., according to results of Swan and Akasaki, if $R = \mathbb{Z}G$, where G is a finite group, then $V^*(R) \setminus V(R)$ is trivial if and only if G is solvable.

In the talk I will briefly summarize some results on $V^*(R)$ but the aim is to present some results on monoids given by isoclasses of pure projective modules over chain rings, that is, rings having the lattices of left and right ideals linearly ordered. These monoids were determined only for nearly simple chain rings and I will discus results we got with Gena Puninski when we tried to calculate these monoids for factors of some rank one chain domains constructed by Brungs and Dubrovin.

Charles University
Sokolovská 83, Prague, Czech Republic
E-mail: prihoda@karlin.mff.cuni.cz

Progress with the Prime Ideal Principle

Manuel Reyes

The Prime Ideal Principle (PIP) for commutative rings (due to T.Y. Lam and the speaker) is a theorem unifying many results in commutative algebra which state that an ideal of a commutative ring that is maximal with respect to *not* possessing certain properties (such as "being proper," "being finitely generated," or "being principal") must be prime. This has subsequently been generalized to a Prime Ideal Principle for right ideals in noncommutative rings, and more recently to two-sided ideals in noncommutative rings.

In this talk, I will survey the various forms of the PIP, including some open questions about the PIP for two-sided ideals. I will also discuss recent applications of the PIP for right ideals when generalizing the notion of "S-finiteness" for a multiplicative set S from commutative to noncommutative algebra.

Department of Mathematics, Bowdoin College
8600 College Station, Brunswick, ME 04011, USA
E-mail: reyes@bowdoin.edu

Almost perfect commutative rings

Luigi Salce

We present the development of the theory of almost perfect commutative rings, from their birth in solving a module theoretical problem, passing to the first basic results for almost perfect domains, then to the blossom of the theory via the tool of cotorsion pairs, and finally arriving to our days with the appearance on the stage of zero-divisors. We will focus on the structural properties of the rings.

Department of Mathematics "Tullio Levi-Civita", Università di Padova
Via Trieste 63, 35131 Padua, Italy
E-mail: salce@math.unipd.it

Non-unique factorizations in bounded hereditary noetherian prime rings

Daniel Smertnig

In a Noetherian ring every non-zero-divisor can be written as a product of atoms (irreducible elements), but usually not uniquely so. Studying non-unique factorizations has a rich history in commutative domains and monoids. More recently, these questions have also been investigated for certain types of noncommutative rings.

In this talk I give an overview over the current status of this area, and in particular present results on bounded hereditary Noetherian prime (HNP) rings. These rings are natural generalizations of Dedekind domains to the noncommutative setting. An important class of examples comes from hereditary orders in central simple algebras over number fields, which themselves can be considered as a noncommutative analogues of rings of algebraic integers.

Under a crucial cancellation property for direct sums of modules, it is possible to construct a transfer homomorphism to a monoid of zero-sum sequences over a subset of an (abelian) class group. This implies that in this case the factorization theory is similar to the one for commutative Dedekind domains.

The proofs are module-theoretic in nature: to study factorizations of an element a in the HNP ring R, one considers composition series of R/aR. A structure theory for finitely generated projective modules for HNP rings, as developed in a recent monograph by Levy and Robson, features prominently.

University of Waterloo
200 University Ave West, Waterloo, ON, Canada N2L 3G1
E-mail: daniel@smertnig.at

Contributed talks

Commutativity theorems in rings with involutions

Adnan Abbasi

The purpose of this paper is to study the commutativity of a ring R with involution $*$, which admits a pair of derivations satisfying certain algebraic identities. In fact certain well known problems like the Herstein problem and the strong commutativity preserving problem have been studied in the setting of rings with involution.

Department of Mathematics, Aligarh Muslim Universitiy
Shiv Puri, AMU Campus, Aligarh, Uttar Pradesh 202001, India
E-mail: adnan.abbasi001@gmail.com

Arithmetic of $\mathcal{O}_0$-modules in the case of small ramification

Sofia Afanaseva

For a specific kind of one-dimensional formal groups over the ring of integers of a local field in the case of small ramification we study the arithmetic of the formal module constructed on the maximal ideal of a local field, containing all the roots of the isogeny. This kind of formal groups is a little broader than Honda groups. The Shafarevich system of generators is constructed. It allows further to find an explicit formulas for the Hilbert symbol for this groups. This problem was completely solved for Honda formal groups, for LubinTate formal groups and in some other cases.

Formal $\mathcal{O}_0$-modules over the ring of integers $\mathcal{O}$ of a local field are formal groups over $\mathcal{O}$ with endomorphism ring including a fixed ring $\mathcal{O}_0$. In the talk we will provide a complete description of the logarithms of all such modules in the case of small ramification.

For a formal group of a special type (π_0-critical), a system of generators of the correspondent $\mathcal{O}_0$-module is constructed.

The present work is supported by the Russian Science Foundation, grant 16-11-10200.

References

[1] T. Honda, *On the theory of commutative formal groups*, J. Math. Soc. Japan **22** (1970), no. 2, 213–246.

[2] S.S. Afanaseva and E.V. Ikonnikova, *Arithmetic of π_0-critical module*, Lobachevskii J. Math. **38** (2017), no. 1, 131–136.

[3] S.S. Afanaseva and R.P. Vostokova, *Logarithms of formal A-modules in the case of small ramification*, Algebra Anal. **27** (2015), no. 6, 6–13.

St. Petersburg State University
St. Petersburg 199178, Russia
E-mail: cheery_sonya@mail.ru

Polynomial functions of the ring of dual numbers modulo m

Amr Ali Abdulkader Al-Maktry

We study the polynomial functions of the ring of dual numbers modulo m, $\mathbb{Z}_m[\alpha]$. Over the ring $\mathbb{Z}_m[\alpha]$, we determine when a polynomial is a null polynomial, when a function is a polynomial function and when a polynomial is a permutation polynomial. In addition to that, for a prime number p, we find the relation between the number of permutation polynomials over the ring $\mathbb{Z}_{p^n}[\alpha]$ and the number of polynomial functions and permutation polynomials over the ring $\mathbb{Z}_{p^n}$. Moreover, when $n \leq p$ explicit formulas for counting polynomial functions and permutation polynomials over $\mathbb{Z}_{p^n}[\alpha]$ are obtained.

Institut für Analysis und Zahlentheorie, TU Graz
Münzgrabenstrasse 37, Graz, Austria
E-mail: amr.al-maktry@tugraz.at

Roots in extensions of domains

Gerhard Angermüller

Etingof, Malcolmson and Okoh have revealed an interesting connection between atomic IDPF-domains and root extensions of domains [1]. In this talk it is shown that this connection is true more generally for monoids, giving new proofs. In a similar way, a result of Jedrzejewicz and Zielinski [2] on root-closed extensions of domains is generalized to monoids; moreover, the monoid methods used here yield a sharpening of this result for domains.

References

[1] P. Etingof, P. Malcolmson, and F. Okoh, *Root Extensions and Factorizations in Affine Domains*, Canad. Math. Bull. **53** (2010), no. 2, 247–255.

[2] P. Jedrzejewicz and J. Zielinski, *Analogs of Jacobian conditions for subrings,* J. Pure Appl. Algebra **221** (2017), no. 8, 2111–2118.

E-mail: `gerhard.angermueller@googlemail.com`

Especially short sequences with full sumset

Austin Antoniou

Let G be a finite abelian group (written additively). A sequence S over G is said to be *zero-sum free* if no nontrivial subsequence of S has sum equal to zero. A longest zero-sum free sequence S over G automatically has the property that every element of $G \setminus \{0\}$ appears as the sum of some subsequence of S. We will explore *shortest* possible zero-sum free sequences with this property.

The Ohio State University
281 W. Lane Avenue, Columbus, OH 43210, USA
E-mail: antoniou.6@osu.edu

On weakly (m,n)-closed ideals and (m,n)-von Neumann regular rings

Ayman Badawi

Let R be a commutative ring with $1 \neq 0$, I a proper ideal of R, and m and n positive integers. In this talk, we define I to be a *weakly (m,n)-closed ideal* if $0 \neq x^m \in I$ for $x \in R$ implies $x^n \in I$, and R to be an *(m,n)-von Neumann regular ring* if for every $x \in R$, there is an $r \in R$ such that $x^m r = x^n$. A number of results concerning weakly (m,n)-closed ideals and (m,n)-von Neumann regular rings are given.

References

[1] D.F. Anderson, A. Badawi, and B. Fahid, *Weakly (m,n)-closed ideals and (m,n)-von Neumann regular rings*, submitted for publication (2017).

[2] D.F. Anderson and A. Badawi, *On (m,n)-closed ideals of commutative rings*, J. Pure Appl. Algebra **16** (2017), no. 1 (21 pages).

[3] D.F. Anderson and A. Badawi, *Von Neumann Regular and Related Elements in Commutative Rings*, Algebra Colloq. **19** (2012), no. 1, 1017–1040.

American University of Sharjah
University City, P.O. Box 26666, Sharjah, United Arab Emirates
E-mail: abadawi@aus.edu

Nonunique factorization in the ring of integer-valued polynomials

Paul Baginski

The ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$ is the set of polynomials with rational coefficients which produce integer values for integer inputs. Specifically,

$$\mathrm{Int}(\mathbb{Z}) = \{f(x) \in \mathbb{Q}[x] \mid \forall n \in \mathbb{Z} f(n) \in \mathbb{Z}\}.$$

$\mathrm{Int}(\mathbb{Z})$ constitutes an interesting example in algebra from many perspectives; for example, it is a natural example of a non-Noetherian ring. It is also a ring with nonunique factorization. It is a finite factorization domain with infinite elasticity. Frisch recently demonstrated that in $\mathrm{Int}(\mathbb{Z})$, you can find an element $f(x)$ that has any factorization lengths you desire and you can even prescribe the number of factorizations of each length. The polynomials constructed in this way have high degree. We give a graded analysis, determining all the possible elasticities and catenary degrees for a polynomial as a function of the degree of the polynomial.

Joint work with: Greg Knapp, Jad Salem, and Gabrielle Scullard.

Fairfield University
1073 North Benson Rd., Fairfield, CT 06824, USA
E-mail: pbaginski@fairfield.edu

On stabilizers of Tits subspaces of $E_6(K)$ for fields K of characteristic two

Mashhour Bani-Ata

The purpose of this talk is to give an elementary construction of certain stabilizers of Tits subspaces of $E_2(K)$ of even characteristics using the notions of generalized quadrangles of type $O_6^-(2)$, cocliques, unipotent radicals and Levi-components.

The talk is based on joint work with Yusuf Alkehzi (ya.alkhezi@paaet.edu.kw).

PAAET-Kuwait
Rd No 103, Kuwait City, Kuwait
E-mail: mashhour.ibrahim@yahoo.com

Dilatations of numerical semigroups

Valentina Barucci

This talk is focused on numerical semigroups and presents a simple construction, that we call dilatation, which, from a starting semigroup S, permits to get an infinite family of semigroups which share several properties with S. The invariants of each semigroup T of this family are given in terms of the corresponding invariants of S and the Apéry set and the minimal generators of T are also described. We also study three properties that are close to the Gorenstein property of the associated semigroup ring: almost Gorenstein, 2-AGL, and nearly Gorenstein properties. More precisely, we prove that S satisfies one of these properties if and only if each dilatation of S satisfies the corresponding one.

All the results of the talk are obtained in a joint work with Francesco Strazzanti.

Dipartimento di Matematica, Sapienza Università di Roma
Piazzale A. Moro 2, 00185 Rome, Italy
E-mail: barucci@mat.uniroma1.it

Nonnoetherian coordinate rings and their noncommutative resolutions

Charlie Beil

I will describe how geometric spaces can be associated to nonnoetherian integral domains with finite Krull dimension. These spaces look like algebraic varieties, but with positive dimensional 'smeared-out' points. In particular, given an algebraic variety X and a finite collection of non-intersecting positive dimensional algebraic sets $Y_i \subset X$, I will show how a nonnoetherian coordinate ring may be constructed whose variety coincides with X except that each Y_i is identified as a distinct closed point. I will also describe noncommutative resolutions of these singularities, and explain how the dimensions of the closed points are captured by the homological properties of the resolutions.

Institute for Mathematics and Scientific Computing, University of Graz
Heinrichstrasse 36, 8010 Graz, Austria
E-mail: charles.beil@uni-graz.at

Pointwise minimal extensions

Paul-Jean Cahen

In a joint work with Gabriel Picavet and Martine Picavet-L'Hermitte, we characterize pointwise minimal extensions of rings, introduced by P.-J. Cahen, D. E. Dobbs and T. G. Lucas in the special context of domains. We show that pointwise minimal extensions are either integral or integrally closed. In the closed case, they are nothing but minimal extensions. In the integral case, there are four cases: either all minimal subextensions are of the same type (ramified, decomposed, or inert) or coexist only ramified and inert minimal subextensions.

Aix-Marseille Université
12 Traverse du Lavoir de Grand Mère, 13100 Aix en Provence, France
E-mail: paul-jean.cahen@univ-amu.fr

On an additive category whose objects have type at most four

Federico Campanini

Two right R-modules A_R and B_R are said to have the same monogeny class (resp. epigeny class) if there exist two monomorphisms (resp. two epimorphisms) $A_R \to B_R$ and $B_R \to A_R$. A module A_R is uniserial if the lattice of its submodules is linearly ordered under inclusion. Finite direct sums of uniserial modules were classified via their monogeny class and their epigeny class by A. Facchini. These two invariants are sufficient because the endomorphism ring of a uniserial module has at most two maximal ideals.

In this talk, we investigate an additive category whose objects are short exact sequences $0 \to A_R \to B_R \to C_R \to 0$ of right R-modules. We describe the endomorphism ring of such objects when A_R and C_R are uniserial modules and the behaviour of these particular short exact sequences as far as their finite direct sums are concerned, proving a weak form of the Krull-Schmidt Theorem. Indeed, we have that finite direct sums of short exact sequences with A_R and C_R uniserial modules are classified via four invariants, which are the natural generalizations of monogeny class and epigeny class. Four invariants are necessary because we are dealing with objects whose endomorphism rings have four maximal ideals (we say that such objects have type four). Finally, we present a slight generalization of these results, introducing a family of additive categories whose objects have type at most $2n$. In this case, finite direct sums of such objects are completely classified via $2n$ invariants.

This talk is based on a joint work with Alberto Facchini.

References

[1] F. Campanini, *On a category of chain of modules whose endomorphism rings have at most 2n maximal ideals*, to appear in Comm. Algebra (2017).

[2] F. Campanini and A. Facchini *On a category of extensions whose endomorphism rings have at most four maximal ideals*, submitted.

[3] A. Facchini, *Krull-Schmidt fails for serial modules*, Trans. Amer. Math. Soc. **348** (1996), 4561–4575.

Department of Mathematics "Tullio Levi-Civita", Università di Padova
Via Trieste 63, 35131 Padua, Italy
E-mail: federico.campanini@phd.unipd.it

From Pólya fields to Pólya groups

Jean-Luc Chabert

A century ago, in 1919, Pólya [1] introduced, for every number field K with ring of integers O_K, the O_K-module formed by the *integer-valued polynomials* on O_K and was interested in fields K for which these O_K-modules admit bases with one polynomial of each degree.

Many years later, in 1982, such fields were called *Pólya fields* by Zantema [2] who undertook a rich study of these fields. In fact, we may forget the integer-valued polynomials thanks to the following characterization: For each prime power p^f, let

$$\Pi_{p^f}(K) = \prod_{M \in \mathrm{Max}(O_K),\, N(M)=p^f} M.$$

Then K is a Pólya field if and only if all the products $\Pi_{p^f}(K)$ are principal ideals.

Several years later, in 1997, the notion of *Pólya group* was introduced: the Pólya group of K is the subgroup of the class group of K generated by the classes of the $\Pi_{p^f}(K)$'s. The Pólya group of K can be considered as a measure of the obstruction for the existence of regular bases, since K is a Pólya field if and only if its Pólya group is trivial. Consequently, each assertion about Pólya groups implies a result concerning Pólya fields.

Surprisingly, the aim of my talk is to consider the reverse implication. More explicitly, I guess that any assertion about Pólya fields is only the evidence of a more general statement about Pólya groups. When studying Pólya fields, they are two cases:

– if K/Q is Galois, we just have to consider the $\Pi_{p^f}(K)$'s where p is ramified;

– if not, on the contrary, we may have to consider just the $\Pi_{p^f}(K)$'s where p is unramified.

After recalling Zantema's results on Pólya fields, I will first make several conjectures on Pólya groups which would imply the previous results. Then, I will try to prove these conjectures, but I do not have enough space to explain how we could do it.

So, the title of my talk could be:

Zantema's paper revisited.

References

[1] G. Pólya, *Über ganzwertige Polynome in algebraischen Zahlkörpern*, J. Reine Angew. Math. **149** (1919), 97–116.

[2] H. Zantema, *Integer valued polynomials over a number field*, Manuscripta Math. **40** (1982), 155–203.

LAMFA, Université de Picardie "Jules Verne"
33 rue Saint Leu, 80034 Amiens, France
E-mail: jean-luc.chabert@u-picardie.fr

UMT-domain property of graded integral domains

Gyu Whan Chang

Let $R = \bigoplus_{\alpha \in \Gamma} R_\alpha$ be an integral domain graded by an arbitrary torsionless grading monoid Γ, $\bar{R}$ be the integral closure of R, H be the set of nonzero homogeneous elements of R, $C(f)$ be the fractional ideal of R generated by the homogeneous components of $f \in R_H$, and $N(H) = \{f \in R \mid C(f)_v = R\}$. Assume that R_H is a UFD. We say that a nonzero prime ideal Q of R is an *upper to zero* in R if $Q = fR_H \cap R$ for some $f \in R$ and that R is a *graded UMT-domain* if each upper to zero in R is a maximal t-ideal. In this talk, we study several ring-theoretic properties of graded UMT-domains. Among other things, we show that if R has a unit of nonzero degree, then R is a graded UMT-domain if and only if every prime ideal of $R_{N(H)}$ is extended from a homogeneous ideal of R, if and only if $\bar{R}_{N(H)}$ is a Prüfer domain, if and only if R is a UMT-domain.

Incheon National University
119 Academy-ro, Yeonsu-gu, Incheon, Republic of Korea
E-mail: whan@inu.ac.kr

The lattice of topologizing filters of rings of continuous functions

Nega Chere

The order dual $[FilR_R]^{du}$ of the set $FilR_R$ of all right topologizing filters on a fixed but arbitrary ring R is a complete lattice ordered monoid with respect to the (order dual) of inclusion and a monoid operation ':' that is, in general, noncommutative. It is known that $[FilR_R]^{du}$ is always left residuated, meaning, for each pair $\mathfrak{F}, \mathfrak{G} \in FilR_R$ there exists a smallest $\mathfrak{H} \in FilR_R$ such that $\mathfrak{H} : \mathfrak{G} \supseteq \mathfrak{F}$, but is not, in general, right residuated (there exists a smallest $\mathfrak{H}$ such that $\mathfrak{G} : \mathfrak{H} \supseteq \mathfrak{F}$). For a commutative ring R for which $[FilR_R]$ is commutative is equivalent to $[FilR_R]^{du}$ both left and right residuated. Moreover, it is shown that a right fully bounded noetherian ring satisfy the two sided residuated property.

The objective of this talk is to investigate the lattice of topologizing filters of rings of continuous real-valued functions. It is shown that if X is finite and completely regular hausdorff space, then $Fil\,C(X)$ is commutative and hence $[Fil\,C(X)]^{du}$ is two-sided residuated. A Tychonoff space X is called an SV-space if for every prime ideal P of the ring $C(X)$ of continuous real-valued functions on X, the ordered integral domain $C(X)/P$ is a valuation ring. It is proved that if X is an SV-space and P is any prime ideal of $C(X)$, then the kernel functor (the topologizing filter) of ideals of $C(X)/P$ is linearly ordered. It is also shown that if X is a complectly regular Hausdorff space, then $Fil\ C(X))$ is linearly ordered if and only if ideals of $C(X)$ are linearly ordered by inclusion.

References

[1] N. Arega and J.E. van den Berg, *Two-sided residuation in the set of topologizing filters on a ring*, Comm. Algebra **45** (2017), no. 3, 992–1011.

[2] L. Gillman and M. Jerison, *Rings of continuous functions*, D. Van Nostrand Company Inc., New York, 1960.

[3] J.S. Golan, *Linear topologies on a ring: An overview*, Pitman Res. Notes Math. Ser. **159**, Longman Scientific & Technical, Harlow, 1987.

[4] C.W. Kohls, *Ideals in rings of continuous functions*, Fund. Math. **45** (1957), 28–50.

[5] M. Henriksen and R. Wilson, *When is $C(X)/P$ a valuation ring for every prime ideal P?*, Topology Appl. **44** (1992), 175–180.

[6] J.E. van den Berg, *When multiplication of topologizing filters is commutative*, J. Pure Appl. Algebra **140** (1999), 87–105.

Addis Ababa University
Addis Ababa, Ethiopia
E-mail: nega.arega@aau.edu.et

Weakly nil-clean index and uniquely weakly nil-clean rings

Andrada Cîmpean

We introduce and study the weakly nil-clean index associated to a ring. We also give some simple properties of this index and show that rings with the weakly nil-clean index 1 are precisely those rings that are abelian weakly nil-clean, thus showing that they coincide with uniquely weakly nil-clean rings. Next, we define certain types of nilpotent elements and weakly nil-clean decompositions by obtaining some results when the weakly nil-clean index is at most 2 and, moreover, we somewhat characterize rings with weakly nil-clean index 2. After that, we compute the weakly nil-clean index for $\mathbb{T}_2(\mathbb{Z}_p), \mathbb{T}_3(\mathbb{Z}_p)$ and $\mathbb{M}_2(\mathbb{Z}_3)$, respectively, as well as we establish a result on the weakly nil-clean index of $\mathbb{M}_n(R)$ whenever R is a ring. Our results considerably extend and correct the corresponding ones from [1].

References

[1] D.K. Basnet and J. Bhattacharyya, Int. Electron. J. Algebra **15** (2014), 145–156.

Faculty of Mathematics and Computer Science, Babes-Bolyai University
Mihail Kogalniceanu 1, 400084 Cluj-Napoca, Romania
E-mail: cimpean_andrada@yahoo.com

Products of idempotent matrices over Prüfer domains

Laura Cossu

A classical problem, studied since the middle of the 1960's, is to characterize integral domains R satisfying the property (ID_n): "Every singular $n \times n$ matrix over R is a product of idempotent matrices." Important results which describe this property inside the class of Bézout domains (cf. [1,2]) motivated a natural conjecture (C), proposed by Salce and Zanardo in [3]: "If an integral domain R satisfies property (ID_2), then it must be a Bézout domain." Unique factorization domains, projective-free domains and PRINC domains (cf. [3]) verify the conjecture. In support of (C) we prove that an integral domain R satisfying (ID_2) must be a Prüfer domain in which every invertible 2×2 matrix is product of elementary matrices. Moreover, we show that a large class of coordinate rings of plane curves and the ring of integer-valued polynomials Int($\mathbb{Z}$) verify (C) in an equivalent formulation.

References

[1] K.P.S. Bhaskara Rao, *Products of idempotent matrices over integral domains*, Linear Algebra Appl. **430** (2009), no. 10, 2690–2695.

[2] T.J. Laffey, *Products of idempotent matrices*, Linear Multilinear Algebra **14** (1983), no. 4, 309-314.

[3] L. Salce and P. Zanardo, *Products of elementary and idempotent matrices over integral domains*, Linear Algebra Appl. **452** (2014), 130–152.

Department of Mathematics "Tullio Levi-Civita", Università di Padova
Via Trieste 63, 35121 Padua, Italy
E-mail: lcossu@math.unipd.it

New results on the Noether number of finite groups

Kálmán Cziszter

The Noether number $\beta(G)$ of a finite group G gives the maximal possible degree of the generators in any ring of polynomial invariants of G. When G is abelian this number is known to coincide with the Davenport constant $\mathsf{D}(G)$ which gives the maximal length of an irreducible zero-sum sequence of elements of G. Geroldinger and Grynkiewicz conjectured that similar relations could hold between $\beta(G)$ an some non-commutative generalisations of $\mathsf{D}(G)$, as well. To verify these conjectures we calculated the quantities in question for all groups of order less than 32. We found a counterexample to one part of the conjectures while the other part was proven to hold in all the considered cases. We also observed that for any proper subgroup $H < G$ the Noether number $\beta(H)$ is strictly smaller than $\beta(G)$. Later we proved that this fact holds in full generality.

References

[1] K. Cziszter, M. Domokos, and I. Szöllősi, *The Noether numbers and the Davenport constants of the groups of order less than* 32, arXiv:1702.02997.

[2] K. Cziszter and M. Domokos, *Lower bounds on the Noether number*, arXiv:1706.03126.

Alfréd Rényi Institute of Mathematics
Reáltanoda utca 13-15, 1053 Budapest, Hungary
E-mail: `cziszter.kalman@gmail.com`

Value semigroups and good semigroups

Marco D'Anna

Good semigroups (in $\mathbb{N}^2$) arise as value semigroups of curve singularities with two branches, but the class of good semigroups is larger than the class of value semigroups. Good semigroups are a natural generalization of numerical semigroups, but one main difference is that they are not finitely generated. This fact implies many problems in the study of good semigroups: for example, while there is a natural generalization of symmetric numerical semigroups, no definition of complete intersection good semigroups has been given. In this talk I will present some new results about the Apéry set of a good semigroup, both in the general and in the symmetric case, showing how to generalize many properties of the Apéry set of a numerical semigroup. In particular, these results could lead to a definition of complete intersection good semigroup (in $\mathbb{N}^2$).

This is a joint work in progress with Lorenzo Guerrieri and Vincenzo Micale.

Dipartimento di Matematica e Informatica, Università degli Studi di Catania
Viale A. Doria 6, Catania, Italy
E-mail: mdanna@dmi.unict.it

On the nilpotency index of nil algebras

Mátyás Domokos

Kaplansky proved in 1946 that finitely generated associative nil algebras (over a field) of bounded nil index are nilpotent. For positive integers n and m denote by $d(n,m)$ the minimal positive integer d such that if we have $x^n = 0$ for all elements x in an associative algebra R generated by m elements, then $x_1 \cdots x_d = 0$ holds for all $x_1, \dots, x_d \in R$. Recent results of Derksen and Makam on generators of rings of matrix invariants together with a theorem of Zubkov from 1996 yield an explicit upper bound on $d(n,m)$ that is polynomial both in n and m. A lower bound for $d(n,2)$ due to Kuzmin is also extended to positive characteristic.

Alfréd Rényi Institute of Mathematics
Reáltanoda utca 13-15, 1053 Budapest, Hungary
E-mail: domokos.matyas@renyi.mta.hu

Ideal factorization in integral domains

Tiberiu Dumitrescu

This talk is about a certain class of integral domains connected to radical factorization domains studied by Vaughan, Yeaghy and Olberding, and Zerlegung Primideale (ZPUI) domains studied by Olberding. For this circle of ideas, an excellent reference is the book "Factoring Ideals in Integral Domains" by Fontana, Houston and Lucas.

Faculty of Mathematics and Computer Science, University of Bucharest
Str. Academiei 14, 010014, Bucharest, Romania
E-mail: tiberiu_dumitrescu2003@yahoo.com, tiberiu@fmi.unibuc.ro

Factorizations of ideals in noncommutative rings similar to factorizations of ideals in commutative Dedekind domains

Alberto Facchini

Dedekind domains were one of the leitmotifs, or, better, one of the propulsive forces, one of the propulsive ideas, at the origins of Commutative Algebra: There is no uniqueness of factorization for non-zero elements, but there is uniqueness of factorization for non-zero ideals. Well, ... for every non-zero ideal I in a Dedekind domain R, the module R/I is direct sum of finitely many uniserial R-modules, and this seems to be the motivation because of which Dedekind domains have such a good behavior as far as product decompositions of ideals is concerned. Thus we have studied the right ideals I in a (non-commutative) ring R for which the right R-module R/I is a direct sum of finitely many uniserial right R-modules. For such a right ideal I, there is a product decomposition of I, which is unique...

References

[1] A. Facchini, *Krull-Schmidt fails for serial modules*, Trans. Amer. Math. Soc. **348** (1996), 4561–4575.

[2] A. Facchini and Z. Nazemian, *Serial factorizations of right ideals*, submitted for publication (2017).

Department of Mathematics "Tullio Levi-Civita", Università di Padova
Via Trieste 63, 35131 Padua, Italy
E-mail: facchini@math.unipd.it

Unions of sets of lengths

Yushuang Fan

For an atomic monoid H and a positive integer $k \in \mathbb{N}^+$, let $\mathscr{U}_k(H)$ denote the set of all $\ell \in \mathbb{N}^+$ with the property that there are atoms $u_1, \ldots, u_k, v_1, \ldots, v_\ell$ such that $u_1 \cdots u_k = v_1 \cdots v_\ell$ (so, $\mathscr{U}_k(H)$ is the union of all sets of lengths containing k).

The Structure Theorem for Unions states that, for all sufficiently large k, the sets $\mathscr{U}_k(H)$ are almost arithmetical progressions with the same difference and global bound. It is well known that it holds true for Krull domains with finite class group and various classes of Noetherian domains that need not be integrally closed.

We present a new approach to the Structure Theorem framed in the language of arithmetic combinatorics, by deriving, for suitably defined families of subsets of the non-negative integers, a characterization of when the Structure Theorem holds.

This abstract approach allows us to verify, for the first time, the Structure Theorem for a variety of possibly non-cancellative monoids, including monoids of (not necessarily invertible) ideals and monoids of modules. In addition, we provide the very first example of a monoid (actually, a Dedekind domain) that does not satisfy the Structure Theorem.

References

[1] Y. Fan, A. Geroldinger, F. Kainrath, and S. Tringali, *Arithmetic of commutative semigroups with a focus on semigroups of ideals and modules*, J. Algebra Appl. **16** (2017), no. 16 (42 pages).

Mathematical College, China University of Geosciences
College Road 29, Haidian District, Beijing, P.R. China
E-mail: `fys@cugb.edu.cn`

Cluster algebras: Factoriality and class groups

Ana Garcia Elsener

Cluster algebras were introduced by Fomin-Zelevinsky in 2002. Their original motivation was coming from canonical bases in Lie Theory. Today the theory of cluster algebras is connected to various fields of mathematics.

Locally acyclic cluster algebras are Krull domains. Hence their factorization theory is determined by their (divisor) class group and the set of classes containing height-1 prime ideals.

Motivated by this, we investigate class groups of cluster algebras. We show that any cluster algebra that is a Krull domain has a finitely generated free abelian class group, and that every class contains infinitely many height-1 prime ideals. For a cluster algebra associated to an acyclic seed, we give an explicit description of the class group in terms of the initial exchange matrix. As a corollary, we reprove and extend a classification of factoriality for cluster algebras of Dynkin type.

The talk is based on joint work with P. Lampe and D. Smertnig.

Institute for Mathematics and Scientific Computing, University of Graz
Heinrichstrasse 36, 8010 Graz, Austria
E-mail: ana.garcia-elsener@uni-graz.at

On the sets of lengths of Puiseux monoids

Felix Gotti

A Puiseux monoid is an additive submonoid of $\mathbb{Q}_{\geq 0}$. We will provide a brief journey through the sets of lengths of non-finitely generated atomic Puiseux monoids. We begin by presenting a BF-monoid with full system of sets of lengths. It is well known that systems of sets of lengths do not characterize numerical monoids. We shall see that systems of sets of lengths also fail to characterize non-finitely generated atomic Puiseux monoids. In a recent paper, Geroldinger and Schmid found the intersection of systems of sets of lengths of numerical monoids. We shall explain how to extend their result to the setting of atomic Puiseux monoids. Finally, we will relate the sets of lengths of the Puiseux monoid $P = \langle 1/p \mid p \text{ is prime} \rangle$ with the Goldbach's conjecture; in particular, we will see that $\mathsf{L}(2)$ is precisely the set of Goldbach's numbers.

References

[1] J. Amos, S.T. Chapman, N. Hine, and J. Paixao, *Sets of lengths do not characterize numerical monoids*, Integers **7** (2007), A50.

[2] A. Geroldinger, *Sets of lengths*, Amer. Math. Monthly **123** (2016), 960–988.

[3] A. Geroldinger and W. Schmid, *A realization theorem for sets of lengths in numerical monoids*, arXiv:1710.04388.

[4] F. Gotti, *On the atomic structure of Puiseux monoids*, J. Algebra Appl. **16** (2017), no. 7 (20 pages).

[5] F. Gotti, *Systems of sets of lengths of Puiseux monoids*, arXiv:1711.06961.

[6] F. Gotti and M. Gotti: *Atomicity and boundedness of monotone Puiseux monoids*, Semigroup Forum **95** (2017), 1–17.

[7] W.A. Schmid, *A realization theorem for sets of lengths*, J. Number Theory **129** (2009), 990–999.

Department of Mathematics, UC Berkeley
970 Evans Hall, Berkeley, CA 94720, USA
E-mail: felixgotti@berkeley.edu

Directed unions of local quadratic and monoidal transforms and GCD domains

Lorenzo Guerrieri

Let $(R, \mathfrak{m})$ be a regular local ring of dimension $d \geq 2$. A local quadratic transform of R is a ring of the form

$$R_1 = R\left[\frac{\mathfrak{m}}{x}\right]_{\mathfrak{m}_1}$$

where $x \in \mathfrak{m}$ is a regular parameter and $\mathfrak{m}_1$ is a maximal ideal of $R[\frac{\mathfrak{m}}{x}]$ containing $\mathfrak{m}$.

Recently, several authors have studied rings of the form $S = \bigcup_{n=0}^{\infty} R_n$, obtained as an infinite directed union of iterated local quadratic transforms of R.

Here we present some results about such rings and we discuss the more general case of local monoidal transforms of R.

The talk is based on joint work with W. Heinzer, B. Olberding, and M. Toeniskoetter.

References

[1] L. Guerrieri, W. Heinzer, B. Olberding, and M. Toeniskoetter, "Directed Unions of Local Quadratic Transforms of Regular Local Rings and Pullbacks", pp. 257–280 in: M. Fontana, S. Frisch, S. Glaz, F. Tartarone, and P. Zanardo (eds.), *Rings, Polynomials, and Modules*, Springer, Cham, 2017.

Università degli Studi di Catania
Viale Andrea Doria 6, 95125 Catania, Italy
E-mail: guelor@guelan.com

Quantum polynomial rings

Ashish Gupta

Quantum polynomial rings are defined using quasi-commuting variables $xy = qyx$. They are of interest due to their links with various other fields including physics. We will describe their matrix representations, infinite dimensional representations, automorphisms, ideal structure, etc.

Ramakrishna Mission Vivekananda Educational and Research Institute
PO Belur Math, Dist Howrah 711202, West Bengal, India
E-mail: a0gupt@gmail.com

A note on direct injective modules

Ashok Ji Gupta

Let R be a commutative ring with unity. A right R-module M is said to be direct injective if, given a direct summand N of M with inclusion $i_N : N \to M$ and a monomorphism $g : N \to M$ there exists $f \in \mathrm{End}_R(M)$ such that $f \circ g = i_N$.

Direct injective modules were introduced by W.K. Nicholson in 1976 as a generalization of quasi-injective modules.

In the talk, we examine properties of direct injective modules in the restricted context of endoregular modules, SSP and SIP modules. In addition, we discuss an equivalent condition for a direct injective module to be divisible, and we show that the endomorphism ring of an R-module is a division ring if and only if M is a direct injective module with the $(*)$ property.

Department of Mathematical Sciences, Indian Institute of Technology
Banaras Hindu University Campus, Varanasi 221005, Uttar Pradesh, India
E-mail: agupta.apm@itbhu.ac.in

On the class group of formal power series rings

Ahmed Hamed

Let $\mathrm{Cl}_t(D)$ denote the t-class group of an integral domain D. P. Samuel has established that if D is a Krull domain then the mapping $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[[X]])$, is injective and if D is a regular UFD, then $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[[X]])$, is bijective. Later, L. Claborn extended this result in case D is a regular Noetherian domain.

In the first part of this talk we prove that the mapping $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[[X]]) : [I] \mapsto [(I.D[[X]])_t]$ is an injective homomorphism and in case of an integral domain D such that each v-invertible v-ideal of D has v-finite type, we give an equivalent condition for $\mathrm{Cl}_t(D) \to \mathrm{Cl}_t(D[[X]])$, to be bijective, thus generalizing the result of Claborn.

In the second part, we define the S-class group of an integral domain D: Let S be a (not necessarily saturated) multiplicative subset of an integral domain D. Following [1], a non-zero fractional ideal I of D is S-principal if there exist an $s \in S$ and $a \in I$ such that $sI \subseteq aD \subseteq I$. The S-class group of D, S-$\mathrm{Cl}_t(D)$, is the group of fractional t-invertible t-ideals of D under t-multiplication modulo its subgroup of S-principal t-invertible t-ideals of D. We generalize some known results developed for the classic contexts of Krull and PvMD domains and we investigate the case of isomorphism S-$\mathrm{Cl}_t(D) \simeq S$-$\mathrm{Cl}_t(D[[X]])$.

References

[1] H. Kim, M.O. Kim, and J.O. Lim, *On S-strong Mori Domains.* J. Algebra **416** (2014), 314–332.

Department of Mathematics, Faculty of Sciences, University of Monastir
Avenue de l'Environment, 5000 Monastir, Tunisia
E-mail: hamed.ahmed@hotmail.fr

Factorization in Prüfer domains

Richard Erwin Hasenauer

We construct a norm on the nonzero elements of a Prüfer domain and extend this concept to the set of ideals of a Prüfer domain. These norms are used to study factorization properties of Prüfer domains.

Northeastern State University
600 N Grand Avenue, Tahlequah, OK 74464, USA
E-mail: hasenaue@nsuok.edu

Monoids of countably generated pure-projective modules

Dolors Herbera

A module is pure-projective if it is isomorphic to a direct summand of a direct sum of finitely presented modules. In general, these modules are direct sums of countably generated modules; hence, to understand their direct sum decomposition there are a couple of reductions we can make: (i) consider countably generated pure-projective modules; (ii) given a countably generated pure-projective module M consider the additive monoid $V^*(M)$ of representatives isomorphism classes of countably generated direct summands of $M^{(\mathbb{N})}$ with the addition induced by the direct sum.

In this talk we want to present the class of monoids that can appear when M is a finitely generated module over a local noetherian ring.

The techniques we use have a somewhat long history. The starting of everything was a result of Příhoda showing that two (non necessarily finitely generated) projective modules are isomorphic iff they are isomorphic modulo the Jacobson radical [4], which opened the way to adapt the existing techniques to study monoids of finitely generated *projective* modules over some classes of rings to the countable case. For noetherian rings, this was completed in [5] and [1]. Some papers by P. Příhoda and G. Puninski [6,7] were showing the way towards the pure-projective case.

In this talk, we will present an ongoing project with Příhoda and Wiegand, in which we are undertaking a systematic study of the commutative noetherian case. We will concentrate on results in [2]: exposing a categorical reformulation for the whole theory of this class of monoids, as well as some realization results for $V^*(M)$ for M a finitely generated module over a commutative noetherian domain of Krull dimension 1. The latter is based in extending the realization techniques of [8] (see also [3]) to the countably generated case.

References

[1] D. Herbera and P. Příhoda, *Big Projective Modules over Noetherian Semilocal Rings*, J. Reine Angew. Math. **648** (2010), 111–148.

[2] D. Herbera, P. Příhoda, and R. Wiegand, *Big pure-projective modules over commutative noetherian rings: comparing with the completion*, preprint (2018).

[3] G.L. Leuschke and R. Wiegand, *Cohen-Macaulay Representations*, Math. Surveys Monogr. **181**, Amer. Math. Soc., 2012.

[4] P. Příhoda, *Projective modules are determined by their radical factors*, J. Pure Appl. Algebra **210** (2007), 827–835.

[5] P. Příhoda, *Fair-sized projective modules*, Rend. Semin. Mat. Univ. Padova **123** (2010), 141–167.

[6] P. Příhoda and G. Puninski, *Classifying generalized lattices: some examples as an introduction*, J. Lond. Math. Soc. **82** (2010), 125–143.

[7] P. Příhoda, G. Puninski and C. Toffalori, *Pure projective torsion free modules over Bass domains*, J. Pure Appl. Algebra **217** (2013), 757–762.

[8] R. Wiegand, *Direct-Sum Decompositions over Local Rings*, J. Algebra **240** (2001), 83–97.

Departament de Matemátiques, Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Spain
E-mail: dolors@mat.uab.cat

From overrings of Prüfer domains to rings in non-integrally closed ring extensions

Ali Jaballah & Mabrouk Ben Nasr

The cardinality of the set of intermediate rings in extensions of integral domains $R \subset S$ has been investigated in several papers by several authors along the last two decades. The main concern has been to compute the number of intermediate rings when this number is finite. The investigations concerning this problem was initiated with the case where the ring R was assumed to be a Prüfer domain with a finite Y-free spectrum and S was assumed to be the field of fractions of R. Then improvements have been gradually obtained in several papers. Recently the case where R is not necessarily integrally closed in S, and where S is not necessarily the field of fractions of R is being studied. We show how these developments were necessary in order to obtain quite general and satisfactory results. We establish several properties relating rings in such extensions to prime ideals. This enable us to establish several equations concerning the cardinality of the set of intermediate rings in the case where some finiteness conditions are satisfied. We also provide examples related to the obtained results.

Department of Mathematics, College of Sciences, University of Sharjah
University City, P.O. Box 27272, Sharjah, United Arab Emirates
E-mail: ajaballah@sharjah.ac.ae

The finitistic weak dimension of a pseudo-valuation domain is at most one

Hwankoo Kim

In this talk, we first characterize the finitistic dimension of a commutative ring in terms of cotorsion theory. Then some characterizations of integral domains with finitistic weak dimension at most one are given. We also characterize the finitistic weak dimension in a special pullback and then obtain, as a corollary, the result that the finitistic weak dimension of a pseudo-valuation domain is at most one. This is a joint work with F. Wang.

Hoseo University
Baebang-up, Asan, Republic of Korea
E-mail: hkkim@hoseo.edu

Polynomial decomposition and linearly recurrent sequences of polynomials

Dijana Kreso

The possible ways of writing a polynomial as a composition of lower degree polynomials were studied by many authors, starting with J.F. Ritt in the 1920's. There are applications to several areas of mathematics. For a sequence of polynomials $(G_n(x))_{n=0}^{\infty}$ in $\mathbb{C}[x]$ satisfying a linear recurrence relation of order $d \geq 2$:

$$G_{n+d}(x) = A_{d-1}(x)G_{n+d-1}(x) + \cdots + A_0(x)G_n(x), \ \ n \in \mathbb{N},$$

determined by $A_0, A_1, \ldots, A_{d-1}, G_0, G_1, \ldots, G_{d-1} \in \mathbb{C}[x]$, one may ask about the properties of $g(x), h(x) \in \mathbb{C}[x]$ such that $G_n(x) = g(h(x))$. In this talk I will present some results on this topic that come from a joint work with Clemens Fuchs and Christina Karolus from University of Salzburg (Austria). Our work was inspired by Zannier's results about lacunary polynomials.

University of British Columbia
1984 Mathematics Road, Vancouver, Canada, BC V6T 1Z2
E-mail: kreso@math.ubc.ca

Delta sets of affine semigroups

David Llena

The contents of this talk are part of joint work with P. García-Sánchez and A. Moscariello [1, 2] (see [3] for related work).

Recently, we have been able to relate the calculation of delta sets of affine semigroups with the computation of a certain Gröbner basis related to the defining ideal of the affine semigroup. This opens a new strategy to determine which sets of integers can be realized as the delta set of an affine semigroup. We solved this realization problem for numerical semigroups whose embedding dimension is three, but using different techniques.

References

[1] P.A. García-Sánchez, D. Llena, and A. Moscariello, *Delta sets for symmetric numerical semigroups with embedding dimension three*, Aequationes Math. **91** (2017), 579–600.

[2] P.A. García-Sánchez, D. Llena, and A. Moscariello, *Delta sets for nonsymmetric numerical semigroups with embedding dimension three*, Forum Math. **30** (2018), no. 1, 15–30.

[3] P.A. Garcá-Sánchez, C. O'Neill, and G. Webb, *On the computation of factorization invariants for affine semigroups*, to appear in J. Algebra Appl.

Departamento de Matemáticas, Universidad de Almeria
E-04120 Almeria, Spain
E-mail: dllena@ual.es

On $*$-clean non-commutative group rings

Yuanlin Li

A ring with involution $*$ is called $*$-clean if each of its elements is the sum of a unit and a projection ($*$-invariant idempotent). In this talk, we discuss the group algebras of the dihedral groups D_{2n}, and the generalized quaternion groups Q_{2n} with standard involution $*$. For the non-semisimple group algebra case, we characterize the $*$-cleanness of RD_{2p^k} with a prime $p \in J(R)$, and RD_{2n} with $2 \in J(R)$, where R is a commutative local ring and $J(R)$ is its Jacobson radical. For the semisimple group algebra case, we investigate when KG is $*$-clean, where K is the field of rational numbers $\mathbb{Q}$ or a finite field F_q and $G = D_{2n}$ or $G = Q_{2n}$.

Brock University
1812 Sir Isaac Brock Way, St. Catharines (ON), Canada, L2S 3A1
E-mail: yli@brocku.ca

Ideal theory of integer-valued polynomials in several variables

Alan K. Loper

The ring $\mathrm{Int}(\mathbb{Z})$ of integer-valued polynomials in one variable over $\mathbb{Z}$ is a Prüfer domain that has been extensively studied. The ring $\mathrm{Int}(\mathbb{Z}^2)$ of integer-valued polynomials over $\mathbb{Z}$ in two variables is not a Prüfer domain and seems not to have been extensively studied. Although it is not Prüfer, it has many Prüfer-like properties that we investigate.

Ohio State University
1179 University Avenue, Newark, OH 43055, USA
E-mail: loper.4@osu.edu

Valuative Marot Rings

Tom Lucas

For a commutative ring R with total quotient ring $T(R)$, R is said to be valuative if for each nonzero $t \in T(R)$ at least one of the extensions $R \subseteq R[t]$ and $R \subseteq R[(R:t)]$ has no proper intermediate rings (where for nonempty $X \subseteq T(R)$, $(R:X) = \{s \in T(R) \mid sX \subseteq R\}$). There are weak and strong versions: R is weakly valuative if for each pair $s,t \in T(R) \setminus \{0\}$ such that $st \in R$, at least one of $R \subseteq R[t]$ and $R \subseteq R[s]$ has no proper intermediate rings; R is strongly valuative if for each nonzero $t \in T(R)$ at least one of the extensions $R \subseteq R[(R:(R:t))]$ and $R \subseteq R[(R:t)]$ has no proper intermediate rings. There are weakly valuative rings that are not valuative, and valuative rings that are not strongly valuative. However, if R is a Marot ring (each regular ideal is generated by regular elements), then all three types are the same. In general, a strongly valuative ring can have infinitely many regular maximal ideals, but a Marot ring that is (weakly) valuative has at most three regular maximal ideals, and only two when it is not integrally closed. In addition, valuative Marot rings can be characterized in much the same way that valuative domains are. For example, if R is an integrally closed Marot ring, then it is valuative if and only if it is a Prüfer ring (each finitely generated regular ideal is invertible) with at most three regular maximal ideals, the set of regular nonmaximal prime ideals is linearly ordered under $\subseteq$ and at most one regular maximal ideal fails to contain each regular nonmaximal prime ideal.

University of North Carolina at Charlotte
9201 University City Blvd, Charlotte, NC 28223, USA
E-mail: tglucas@uncc.edu

The Sum-Essential Left Ideal Graph of Rings

Ali Majidinya

Let R be a unital ring which is not necessarily commutative. The sum-essential left ideal graph of R, denoted by ε_R, is the graph whose vertices are all the nontrivial left ideals of R and where two distinct left ideals I and J are adjacent if $I+J$ is an essential left ideal of R. It is shown that ε_R is a connected graph for every ring R with at least one nontrivial left ideal and $\mathrm{diam}(\varepsilon_R) \leq 3$. It is shown that if ε_R is a tree, then ε_R is a complete graph of order 2 or a star graph. We prove that the girth of ε_R belongs to the set $\{3, \infty\}$. We find a sharp lower bound for the independence number of ε_R, in terms of the uniform dimension (Goldie dimension) of R as a left R-module. When R is simple, we obtain the exact value of the independence number of the graph ε_R. Some other properties of ε_R are used for investigating the structure of the ring R.

References

[1] J. Amjadi, *The essential ideal graph of a commutative ring*, Asian-Eur. J. Math. **11** (2018), no. 2, DOI: 10.1142/S1793557118500584.

[2] S. Akbari, D. Kiani, F. Mohammadi, and S. Moradi, *The total graph and regular graph of a commutative ring*, J. Pure Appl. Algebra **213** (2009), 2224–2228.

[3] S. Akbari and A. Mohammadian, *On zero-divisor graphs of finite rings*, J. Algebra **314** (2007), 168–184.

[4] D.F. Anderson and A. Badawi, *The total graph of a commutative ring*, J. Algebra **320** (2008), no. 7, 2706–2719.

[5] K. Cvetko-Vah and D. Dolzan, *Indecomposability graphs of rings*, Bull. Aust. Math. Soc. **77** (2008), 151–159.

[6] D. Lu and T. Wu, *On bipartite zero-divisor graphs*, Discrete Math. **309** (2009), 755–762.

[7] S. Redmond, *On zero-divisor graphs of small finite commutative rings*, Discrete Math. **307** (2007), 1155–1166.

[8] H.J. Wang, *Graphs associated to co-maximal ideals of commutative rings*, J. Algebra **320** (2008), 2917–2933.

[9] T. Wu, Q. Liu, and L. Chen, *Zero-divisor semigroups and refinements of a star graph*, Discrete Math. **309** (2009), 2510–2518.

Salman Farsi University of Kazerun
Taleghani street, Kazerun, Fars, Iran
E-mail: ali.majidinya@gmail.com, ali.majidinya@kazerunsfu.ac.ir

Radical factorization in rings with zero-divisors

Tusif Ahmed Malik

In this talk I present my work on radical factorization with my supervisor T. Dumitrescu. We extend the work of Vaughan, Yeagy and Olberding on integral domains with radical factorization to the case of rings with zero-divisors. We study the rings in which every ideal (resp. every regular ideal) can be written as a finite product of radical ideals.

References

[1] M.T. Ahmed and T. Dumitrescu, *SP-rings with zero-divisors*, Comm. Algebra **45** (2017), 4435–4443.

[2] R. Gilmer, *Multiplicative ideal theory*, Queen's Papers Pure Appl. Math. **90**, Queen's University, Kingston (CA), 1992.

[3] B. Olberding, "Factorization into radical ideals", pp. 363–377 in: S.T. Chapman (ed.), *Arithmetical properties of commutative Rings and monoids*, Lect. Notes Pure Appl. Math. **241**, Chapman & Hall, 2005.

[4] A. Reinhart, *Radical factorial monoids and domains*, Ann. Sci. Math. Québec **36** (2012), 193–229.

[5] O.A. Heubo-Kwegna, B. Olberding, and A. Reinhart, *Group-theoretic and topological invariants of completely integrally closed Prüfer domains*, J. Pure Appl. Algebra **220** (2016), 3927–3947.

[6] N. Vaughan and R. Yeagy, *Factoring ideals into semiprime ideals*, Canad. J. Math. **30** (1978), 1313–1318.

Abdus Salam School of Mathematical Sciences, GC University
68-B New Muslim Town, Lahore, Pakistan
E-mail: tusif.ahmad92@gmail.com

Commutative rings with two-absorbing factorizations (TAF-rings)

Muzammil Mukhtar

In this talk, I will present my work done with my PhD supervisor Dr. Tiberiu Dumitrescu. I will use the concept of 2-absorbing ideal introduced by Aymen Badawi to discuss those commutative rings in which every ideal can be written as a finite product of 2-absorbing ideals. I call them TAF-rings. I will present some properties and show that any TAF-ring has dimension at most one.

References

[1] A. Badawi, *On 2-absorbing ideals of commutative rings*, Bull. Austral. Math. Soc. **75** (2007), 417–429.

[2] R. Gilmer, *Multiplicative ideal theory*, Queen's Papers Pure Appl. Math. **90**, Queen's University, Kingston (CA), 1992.

[3] M. Mukhtar, M.T. Ahmed, and T. Dumitrescu, *Commutative rings with two-absorbing factorization*, Comm. Algebra (DOI:10.1080/00927872.2017.1332202).

Abdus Salam School of Mathematical Sciences, GC University
68-B New Muslim Town, Lahore, Pakistan
E-mail: muzammilmukhtar3@gmail.com, muzammilmukhtar@sms.edu.pk

Non-unique factorizations in rings of integer-valued polynomials on certain Dedekind domains

Sarah Nakato

For a domain D with quotient field K, the ring of integer-valued polynomials on D,

$$\mathrm{Int}(D) = \{f \in K[x] : f(D) \subseteq D\},$$

is in general not a unique factorization domain.

In this talk, we discuss non-unique factorizations in $\mathrm{Int}(D)$ where D is a Dedekind domain whose maximal ideals are all of finite index, and such that D has infinitely many maximal ideals, but at most finitely many for each individual index.

We present two main results. First, for any finite multiset N of natural numbers greater than 1, there exists a polynomial $f \in \mathrm{Int}(D)$ which has exactly $|N|$ essentially different factorizations of the prescribed lengths. In particular, this implies that every finite non-empty set N of natural numbers greater than 1 occurs as a set of lengths of a polynomial $f \in \mathrm{Int}(D)$. Second, we show that the multiplicative monoid of $\mathrm{Int}(D)$ is not a transfer Krull monoid.

Graz University of Technology
Kopernikusgasse 24, 8010 Graz, Austria
E-mail: snakato@tugraz.at

Counting distinct fuzzy subgroups of dihedral groups D_{2p^sq}

Abdulhakeem Olayiwola

Counting distinct fuzzy subgroups of dihedral groups is a fundamental combinatorics problem. Without any equivalence relation the number of distinct fuzzy subgroups of any finite group is infinite. This work was therefore devoted to deriving explicit formula counting distinct fuzzy subgroups of the dihedral group D_{2p^sq} with respect to a new equivalence relation $\approx$ known in literature.

Sule Lamido University
Kafin Hausa, Jigawa, Nigeria
E-mail: olayiwola.a@jsu.edu.ng

A type of generalized factorizations

Reyes Matiel Ortiz-Albino

The notion of a τ-factorization or τ-products in the general theory of (nonatomic) factorization was defined in 2006. Since, several results have been done in general; including other type of generalizations. This talk will focus more on the theory of τ-factorization, when τ is an equivalence relation on the nonzero nonunit integers. We will recall the definition and several results given by Anderson and Frazier, then will show how the setting changes by considering an equivalence relation. Among the results the associate preserving closure of an equivalence relation and its utility on $\tau_{(n)}$-number theory.

References

[1] D.D. Anderson, D.F. Anderson, and M. Zafrullah, *Factorization in integral domains*, J. Pure Appl. Algebra **69** (1990), no. 1, 1–19.

[2] D.D. Anderson and A.M. Frazier, *On a general theory of factorization in integral domains*, Rocky Mountain J. Math. **41** (2011), no. 3, 663–705.

[3] C.A. Serna, *Factorizaciones donde cada factor de un elemento pertenece a una clase de equivalencia*, Master's Thesis, University of Puerto Rico-Mayagüez (2014).

[4] R.M. Barrios, *A type of maximum common factor*, Master's Thesis, University of Puerto Rico-Mayagüez (2016).

[5] C.A. Molina, *On the number of $\tau_{(n)}$-factors*, Master's Thesis, University of Puerto Rico-Mayagüez (2016).

University of Puerto Rico at Mayagüez
Mayagüez, 00682, Porto Rico 00681-9018
E-mail: reyes.ortiz@upr.edu

On Hilbert and Goldman Rings Issued from Amalgamated Algebras

Omar Ouzzaouit

Let A and B be two rings, J an ideal of B, and $\varphi : A \longrightarrow B$ a ring homomorphism. The ring

$$A \bowtie^{\varphi} J := \{(a, \varphi(a) + j) : a \in A \text{ and } j \in J\}$$

is called *the amalgamation of A with B along J with respect to φ*. It was proposed by D'Anna et al. ([4] and [5]) as a generalization of the Nagata idealization.

In this talk, we present some recent developments in the study of algebraic properties of the amalgamated algebras. In particular, we give necessary and sufficient conditions for $A \bowtie^{\varphi} J$, and some related constructions, to be either a Hilbert ring, a G-domain, or a G-ring.

Besides, we investigate the transfer of the G-property among pairs of rings sharing an ideal. Then, for $A \subseteq B$ a couple of rings, we establish necessary and sufficient conditions for (A, B) to be a G-ring pair (that is, each intermediate ring $A \subseteq R \subseteq B$ is a G-ring). That leads us to generalize a theorem of D. Dobbs [6], characterizing G-domain pairs, to pairs of rings with zero divisors. Also we provide original illustrating examples.

References

[1] J.C. Adams, *Rings with a finitely generated total quotient ring*, Canad. Math. Bull. **17** (1974), no. 1, 1–4.

[2] D.F. Anderson, D.E. Dobbs, and M. Fontana, *Hilbert rings arising as pullbacks*, Canad. Math. Bull. **35** (1992), no. 4, 431–438.

[3] D.D. Anderson and M. Winders, *Idealization of a Module*, J. Commut. Algebra **1** (2009), no. 1, 3–56.

[4] M. D'Anna and M. Fontana, *An amalgamated duplication of a ring along an ideal: The basic properties*, J. Algebra Appl. **6** (2007), no. 3, 433–459.

[5] M. D'Anna, C.A. Finnocchiaro, and M. Fontana, "Amalgamated algebras along an ideal", pp. 155–172 in: M. Fontana, S.-E. Kabbaj, B. Olberding, and , I. Swanson, *Commutative Algebra and its Applications: Proceedings of the Fifth International Fez Conference on Commutative Algebra and Applications, Fez, Morocco, June 23-28, 2008*, de Gruyter, Berlin, 2009.

[6] D. Dobbs, "G-domain pairs", pp. 71–75 in: A. Badawi, *Trends in Commutative Rings Research*, Nova Science Pub. Inc., 2003.

Faculté des Sciences Semlalia, Université Cadi Ayyad
Boulevard Prince Moulay Abdellah, Marrakech 40000, Marocco
E-mail: ouzzaouitomar@gmail.com

Fully-invariant Semi-Injective Modules

Manoj Kumar Patel

In this talk we relate Fully-invariant (FI) Semi-Injective modules with Hopfian and co-Hopfian modules. Moreover, we discuss various properties of FI Semi-Injective modules such as the summand intersection property (SIP), the summand sum property (SSP), substitution, cancellation, internal cancellation, and direct finiteness.

References

[1] G.F. Birkenmeier, B.J. Muller, S.T. Rizvi, *Modules in which Every Fully Invariant Submodule is Essential in a Direct Summand*, Comm. In Algebra **30** (2002), no. 3, 1395-1415.

[2] J. Clark, C. Lomp, N. Vanaja, and R. Wisbauer, *Lifting Modules. Supplements and projectivity in module theory*, Frontiers in Math. Boston, Birkhauser, (2006).

Department of Mathematics, National Institute of Technology at Nagaland
Chumukedima, Dimapur, Nagaland 797103, India
E-mail: mkpitb@gmail.com

On the metric dimension of a zero-divisor graph associated with a commutative ring

Shariefuddin Pirzada

Let R be a commutative ring with unity 1 and let $G(V, E)$ be a simple graph. In this talk, we discuss the metric dimension in zero-divisor graphs associated with commutative rings. We show that for a given rational $q \in (0, 1)$ there exists a finite graph G such that $\dim_M(G)/|V(H)| = q$, where H is any induced connected subgraph of G. We provide a metric dimension formula for a zero-divisor graph $\Gamma(R \times \mathbb{F}_q)$ and give metric dimension of the zero-divisor graph $\Gamma(R_1 \times R_2 \times \cdots \times R_n)$, where $R_1, R_2, \cdots, R_n$ are n finite commutative rings with each having unity 1 and none of $R_1, R_2, \ldots, R_n$ is isomorphic to the Boolean ring $\prod_{i=1}^{n} \mathbb{Z}_2$. We also discuss the metric dimension of the Cartesian product of zero-divisor graphs.

We refer to [1] and [2] for related work and background.

References

[1] S. Pirzada, R. Raja, and S. Redmond, *Locating sets and numbers of graphs associated to commutative rings*, J. Algebra Appl. **13** (2014), no. 7 (18 pages).

[2] R. Raja, S. Pirzada, and S.P. Redmond, *On locating numbers and codes of zero divisor graphs associated with commutative rings*, J. Algebra Appl. **15** (2016), no. 1 (22 pages).

[3] S. Pirzada and R. Raja, *On the metric dimension of a zero divisor graph*, Comm. Algebra, **45** (2017), no. 4, 1399–1408.

Department of Mathematics, University of Kashmir
Hazratbal, Srinagar, Kashmir 190006, India
E-mail: pirzadasd@kashmiruniversity.ac.in

Cosilting Modules

Flaviu Pop

The notion of cosilting module was recently introduced in [3], and independently in [6], as a generalization of the notion of cotilting module and it is defined as the categorical dual of the notion of silting module (see [1]). In this talk, we present several characterizations of these modules and discuss their connections with silting modules. We show that Bazzoni's theorem about the pure-injectivity of cotilting modules (see [2]) is also valid for cosilting modules. Moreover, the characterization of cosilting modules in terms of two-term cosilting complexes is given (see [4]). We also give a Cosilting Theorem induced by a finitely cosilting bimodule (see [5]).

References

[1] L. Angeleri Hügel, F. Marks, and J. Vitória, *Silting modules*, Int. Math. Res. Not. IMRN **2016** (2016), no. 4, 1251–1284.

[2] S. Bazzoni, *Cotilting modules are pure-injective*, Proc. Amer. Math. Soc. **131** (2003), no. 12, 3665–3672.

[3] S. Breaz and F. Pop, *Cosilting modules*, Algebr. Represent. Theory **20** (2017), no. 5, 1305–1321.

[4] F. Pop, *A note on cosilting modules*, J. Algebra Appl. **16** (2017), no. 11 (11 pages).

[5] F. Pop, *Finitely cosilting modules*, submitted for publication (2017).

[6] P. Zhang and J. Wei, *Cosilting complexes and AIR-cotilting modules*, J. Algebra **491** (2017), 1–31.

Faculty of Mathematics and Computer Science, Babes-Bolyai University
Mihail Kogalniceanu 1, 400084 Cluj-Napoca, Romania
E-mail: flaviu.v@gmail.com

Perinormal rings with zero-divisors

Anam Rani

In their recent papers: J. Algebra 451 (2016) and arXiv:1511.06473v2 [math.AC], 29 Apr 2016, N. Epstein and J. Shapiro introduced and studied perinormal domains, that is, domains A whose overrings satisfying going down over A are flat A-modules. We study the perinormal concept in the setup of rings with zero-divisors. We extend several results from the case of perinormal domains, e.g., we prove that a Krull ring is perinormal.

The talk is based on joint work with Tiberiu Dumitrescu.

University of Central Punjab
1, Khayaban-e-Jinnah Road, Johar Town, Lahore, Pakistan
E-mail: anamrane@gmail.com

A class of pinched domains with finitely many overrings

Sharif ur Rehman

As an extension of the class of Dedekind domains, we have introduced the class of Globalized multiplicatively pinched-Dedekind domains (GMPD domains). In this talk I will discuss the properties of GMPD domains having only finitely many overrings. I will also discuss results that give the number of overrings of any GMPD domain that has only finitely many overrings.

COMSATS Institute of Information Technology
Kamra Rd, Attock 43600, Pakistan
E-mail: shafiq@ciit-attock.edu.pk, shafiq_ur_rahman2@yahoo.com

---

Radical factorization in ideal systems of monoids

Andreas Reinhart

Let H be a commutative cancellative multiplicative monoid with a zero element and r a finitary ideal system on H. We say that H is an r-SP-monoid if every r-ideal of H is a finite r-product of radical r-ideals of H. Furthermore, H is called an r-almost Dedekind monoid if all localizations of H at r-maximal r-ideals are discrete valuation monoids. In this talk we discuss some recent results concerning radical r-factorizations of r-ideals. We present some new characterizations of r-almost Dedekind r-SP-monoids. One of these characterizations is, for instance, that H is an r-almost Dedekind r-SP-monoid if and only if the radical of every nonzero r-finitely generated r-ideal of H is r-invertible. Moreover, we provide some nice descriptions of w-SP-monoids. In particular, we show that H is a w-SP-monoid if and only if the radical of every principal ideal of the monoid of w-invertible w-ideals of H is principal.

Department of Mathematical Sciences, New Mexico State University
1290 Frenger Mall, Las Cruces, New Mexico 88003-8001, USA
E-mail: andreas.reinhart@uni-graz.at

On extremal product-one free sequences and weighted Davenport constants

Sávio Ribas

Let G be a finite group, written multiplicatively. The *small Davenport constant* of G is the smallest positive integer $d(G)$ such that every sequence of G with $d(G)$ elements has a non-empty subsequence with product 1 in some order. Let $C_q \rtimes_s C_m$ be the metacyclic group where q is a prime and $ord_q(s) = m$, D_{2n} be the dihedral group of order $2n$, $C_n \rtimes_s C_2$ be the metacyclic group where $s \not\equiv \pm 1 \pmod n$, and Q_{4n} be the dicyclic group of order $4n$. J.J. Zhuang & W. Gao [5] showed that $d(D_{2n}) = d(C_n \rtimes_s C_2) = n + 1$ and J. Bass [1] showed that $d(C_q \rtimes_s C_m) = m + q - 1$ and $d(Q_{4n}) = 2n + 1$.

If G is an abelian group, the *A-weighted Davenport constant* of G, $D_A(G)$, is the smallest positive integer d such that every sequence $x_1, \dots, x_d$ of G has a non-empty subsequence $(x_{j_i})_i$ such that $\sum_{i=1}^t \varepsilon_i x_{j_i} = 0$ for some $\varepsilon_i \in A$.

During the talk, we will present the main ideas for the proof of the theorems that give explicit characterizations of all sequences S of G such that $|S| = d(G) - 1$ and S is free of subsequences whose product is 1, where G is equal to $C_q \rtimes_s C_m$, D_{2n}, $C_n \rtimes_s C_2$ and Q_{4n}, such as done in [2], [3] and [4]. For this, we need the values of $D_A(\mathbb{Z}_n)$ for some weight sets A.

This is a joint work with F.E. Brochero Martínez (fbrocher@mat.ufmg.br).

References

[1] J. Bass, *Improving the Erdös-Ginzburg-Ziv theorem for some non-abelian groups*, J. Number Theory **126** (2007), 217–236.

[2] F.E. Brochero Martínez and S. Ribas, *Extremal product-one free sequences in $C_q \rtimes_s C_m$*, submitted for publication (2017). Available at: https://arxiv.org/pdf/1610.09870.pdf.

[3] F.E. Brochero Martínez and S. Ribas, *Extremal product-one free sequences in Dihedral and Dicyclic Groups*, Discrete Math. **341** (2018), no. 2, 570–578.

[4] F.E. Brochero Martínez and S. Ribas, *The $\{1, s\}$-weighted Davenport constant in $\mathbb{Z}_n$ and an application in an inverse problem*, in preparation (2018).

[5] J.J. Zhuang and W. Gao, *Erdös-Ginzburg-Ziv theorem for dihedral groups of large prime index*, European J. Combin. **26** (2005), 1053–1059.

Instituto Federal de Educação, Ciência e Tecnologia de Minas Gerais (IFMG) - Campus Ouro Preto
Rua Pandiá Calógeras, 898 - Bauxita, Ouro Preto (MG), Brazil
E-mail: savio.ribas@ifmg.edu.br

J-ideals of matrices over PIDs

Roswitha Rissner

Given a square matrix B over a commutative ring R and an ideal J of R, the J-ideal is the ideal consisting of the polynomials $f \in R[X]$ such that all entries of $f(B)$ are in J. J-ideals arise naturally in the study of integer-valued polynomials on a single matrix.

If the underlying ring is a principal ideal domain it suffices to determine a finite number of polynomials in order to describe all J-ideals of B. In this talk we discuss new results concerning certain generating sets of J-ideals.

Alpen-Adria-Universität Klagenfurt
Universitätsstrasse 65-67, Klagenfurt, Austria
E-mail: roswitha.rissner@gmx.at

On strongly primary monoids and domains

Moshe Roitman

This talk is based on a joint work with Alfred Geroldinger on strongly primary monoids and domains. Among other results, we proved that a strongly primary domain is locally tame, thus answering in the affirmative Geroldinger's question whether a one-dimensional local Mori domain is locally tame (Problem 38 in [1]). Having established local tameness we show that every one-dimensional local Mori domain has finite catenary degree and all sets of lengths are almost arithmetical progressions with global bounds.

References

[1] P.-J. Cahen, M. Fontana, S. Frisch, and S. Glaz, "Open problems in commutative ring theory", pp. 353–375 in: M. Fontana, S. Frisch, and S. Glaz (eds.), *Commutative Algebra - Recent Advances in Commutative Rings, Integer-valued Polynomials, and Polynomial Functions*, Springer, New York, 2014.

Department of Mathematics, University of Haifa
Mount Carmel, Haifa 31905, Israel
E-mail: mroitman@math.haifa.ac.il

Algorithm for primary submodule decomposition without producing intermediate redundant components

Afshan Sadiq

The aim of this talk is to present an algorithm for the primary decomposition of a submodule $\mathbf{N}$ of the free module $\mathbb{Q}[x_1, \ldots, x_n]^s$. I will use for this purpose the algorithms for primary decomposition of ideals in polynomial rings. I will generalize the method of Kawazoe and Noro to primary decomposition for submodules of free modules.

References

[1] E.W. Rutman, *Gröbner bases and primary decomposition of modules*, J. Symbolic Comput. **14** (1992), 483–503.

[2] N. Idrees, *Algorithms for primary decomposition of modules*, Studia Sci. Math. Hungar. **48** (2011), no. 2, 227–246.

[3] T. Shimoyama and K. Yokoyama, *Localization and Primary Decomposition of Polynomial Ideals*, J. Symbolic Comput. **22** (1996), 247–277.

Abdus Salam School of Mathematical Sciences, GC University
68-B New Muslim Town, Lahore, Pakistan
E-mail: afshansadiq6@gmail.com

"Every" set is a set of lengths of a numerical monoid

Wolfgang A. Schmid

Let H denote an additive monoid, that is, a commutative, cancellative semi-group with neutral element. One says that H is atomic if each non-invertible element is the sum of finitely many irreducible elements.

For $a \in H$ with $a = u_1 + \cdots + u_k$ where u_i is irreducible one calls k a length of a and one denotes by $\mathsf{L}(a)$ the set of all k that are a length of a. For an invertible element a one sets $\mathsf{L}(a) = \{0\}$.

One sees directly that if a set of lengths contains 0 then it equals $\{0\}$ and if it contains 1 then it equals $\{1\}$. All other sets of lengths are subsets of $\mathbb{N}_{\geq 2}$. Moreover, for many types of monoids that are commonly investigated sets of lengths turn out to be finite sets.

The question then arises whether, for a particular type of monoid, there are any further global restrictions on sets of lengths, or whether instead every finite subset of integers greater than or equal to two arises as a set of lengths of some monoid of this type.

An affirmative answer to this question is well-known for monoids of zero-sum sequences over finite abelian groups.

The present talk presents an analogous result for numerical monoids, obtained recently in joint work with A. Geroldinger. A numerical monoid is a submonoid of $(\mathbb{N}_0, +)$ with finite complement.

Theorem. For L a finite subset of $\mathbb{N}_{\geq 2}$ there exists some numerical monoid H_L and some element $a_L \in H_L$ such that $\mathsf{L}(a_L) = L$.

Time permitting refinements and related questions will be discussed as well.

References

[1] A. Geroldinger and W.A. Schmid, *A realization theorem for sets of lengths in numerical monoids*, Forum Math., to appear.

LAGA, Université Paris 8
2, rue de la Liberté, 93200 Saint-Denis, France
E-mail: schmid@math.univ-paris13.fr

A Factorization Theory for some Free Fields

Konrad Schrempf

Although in general there is no meaningful concept of factorization in fields, that in free associative algebras (over a *commutative* field) can be extended to their respective free field (universal field of fractions) on the level of *minimal linear representations*. We establish a factorization theory by providing an alternative definition of left (and right) divisibility based on the *rank* of an element and show that it coincides with the "classical" left (and right) divisibility for non-commutative polynomials. Additionally we present an approach to factorize elements, in particular *rational formal power series*, into their (generalized) atoms. The problem is reduced to solving a system of polynomial equations with *commuting* unknowns.

Graz University of Technology
Steyrergasse 30/III, 8910 Graz, Austria
E-mail: schrempf@math.tugraz.at

Jaffard families and extension of star operations

Dario Spirito

The idea of extending a finite-type star operation on an integral domain D to a localization of D has been introduced by Houston, Mimouni and Park [1] to study integral domain with a low number of star operations. We generalize this concept to star operation not necessarily of finite type and to flat overrings of D, and use it to study the relationship between the set $\mathrm{Star}(D)$ of the star operations on D and the sets of the star operations on the flat overrings of D. In particular, if Θ is a *Jaffard family* of D (a family of flat overrings with strong independence properties) then the concept of extension gives a way to decompose $\mathrm{Star}(D)$ into a product $\prod_{T\in\Theta}\mathrm{Star}(T)$, which preserves the main properties of the star operations.

When D is a Prüfer domain, this decomposition can also be complemented by the possibility of relating the set of semistar operations on D with the set of semistar operations on D_P and D/P (where P is a prime in the Jacobson radical of D), and by the possibility of building semistar operations on D by "gluing" star operations on overrings of D. In particular, this gives an algorithm to study star operations on D by applying repeatedly the processes of extension, quotient and gluing, and can be used to show that (when D is a Prüfer domain with finite spectrum) the sets of star and semistar operations on D are uniquely determined by some geometric data (the homeomorphically irreducible tree associated to $\mathrm{Spec}(D)$) and by some algebraic data (how many primes are locally principal and how many are idempotent).

References

[1] Evan G. Houston, A. Mimouni, and M.H. Park, *Integral domains which admit at most two star operations*, Comm. Algebra **39** (2011), no. 5, 1907–1921.

Università degli Studi "Roma Tre"
Largo San Leonardo Murialdo, 1, 00146 Rome, Italy
E-mail: spirito@mat.uniroma3.it

Integer-valued polynomial over quaternion algebras

Francesca Tartarone

Let $\mathbb{H}_\mathbb{Z}$ and $\mathbb{H}_\mathbb{Q}$ denote, respectively, the algebras over $\mathbb{Z}$ and $\mathbb{Q}$ generated by the unit elements $1, \mathbf{i}, \mathbf{j}, \mathbf{k}$ (with $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$ and $\mathbf{ij} = \mathbf{k} = -\mathbf{ij}, \mathbf{jk} = \mathbf{i} = -\mathbf{kj}, \mathbf{ki} = \mathbf{j} = -\mathbf{ik}$).

In [1], N. Werner studied the set $\mathrm{Int}(\mathbb{H}_\mathbb{Z})$ of integer-valued polynomials over $\mathbb{H}_\mathbb{Z}$ with coefficients in $\mathbb{H}_\mathbb{Q}$, that is,

$$\mathrm{Int}(\mathbb{H}_\mathbb{Z}) = \{f \in \mathbb{H}_\mathbb{Q}[X] : f(\mathbb{H}_\mathbb{Z}) \subseteq \mathbb{H}_\mathbb{Z}\}.$$

After the (non trivial) proof that $\mathrm{Int}(\mathbb{H}_\mathbb{Z})$ is a noncommutative ring, Werner investigated the ideal structure of this ring, describing some prime ideals above the zero ideal and the maximal ideals of $\mathbb{H}_\mathbb{Z}$.

Moving from this ideas, A. Cigliola, K.A. Loper and N. Werner focused on similar problems in a different setting [2]: the set of integer split quaternions $\mathbb{P}_\mathbb{Z}$.

Given a commutative ring R, $\mathbb{P}_R$ is the (noncommutative) R-algebra generated by the four unit elements $1, \mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ with the relations

$$-\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = 1 .$$

Formally we obtain elements of the form $\mathbb{Q} = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}$, with integer coefficients a, b, c, and d. One of the main differences between $\mathbb{H}_\mathbb{Z}$ and $\mathbb{P}_\mathbb{Z}$ is that $\mathbb{P}_\mathbb{Z}$ is not an integral domain.

In a paper (in progress) with A. Cigliola, we study localization properties and the prime spectrum of the integer-valued polynomial ring

$$\mathrm{Int}(\mathbb{P}_\mathbb{Z}) := \{f \in \mathbb{P}_\mathbb{Q}[x]; f(\mathbb{P}_\mathbb{Z}) \subseteq \mathbb{P}_\mathbb{Z}\}.$$

In particular, we use a matrix representation for $\mathbb{P}_\mathbb{Z}$ to investigate the ideals of $\mathrm{Int}(\mathbb{P}_\mathbb{Z})$.

References

[1] N.J. Werner, *Integer-valued polynomials over quaternion rings*, Ph.D. Thesis, The Ohio State University, 2010.

[2] A. Cigliola, K.A. Loper, and N. Werner, *Split quaternions and integer-valued polynomials*, Comm. Algebra **43** (2015), no. 1, 182–196.

Università degli Studi "Roma Tre", Dipartimento di Matematica e Fisica
Largo San Leonardo Murialdo, 1, 00146 Rome, Italy
E-mail: tfrance@mat.uniroma3.it

Decomposition over a division algebra and Ramanujan complexes

Uzi Vishne

Ramanujan graphs, which are finite regular graphs with optimal spectral properties, found a wide range of applications in combinatorics. Their construction is based on representation theory of local groups and the arithmetic of quaternion algebras. Similarly, the construction of Ramanujan complexes has led to growing interest in higher dimensional combinatorics, and to numerous applications. We will describe the construction, emphasizing the role of decomposition of polynomials over a division algebra in the explicit description of the Bruhat-Tits building, whose quotients are the desired complexes.

Bar-Ilan University
Ramat-Gan, 52900 Israel
E-mail: vishne@math.biu.ac.il

Formal modules in constructible class field theory

Sergey Vostokov & Ilia Nekrasov

We are mainly focused on the formal modules $F(\mathfrak{m}_K)$, where $\mathfrak{m}_K$ is a maximal ideal of some local field K, and $F = F(X,Y)$ is a formal group law defined over the ring of integers of K. This modules is of high importance especially because it is domain of a generalized reciprocity pairing. And the explicit construction of the last one, also known as a Hilbert pairing, is a main problem in constructible class field theory.

In the talk we will present the structure results for such formal modules as a Galois modules, including the recent results such as

- explicit description of cohomology group $H^1(Gal(L/K), F(\mathfrak{m}_L))$ for an arbitrary extension L/K of local fields and some formal group laws F;
- explicit "generators–relations" description of the Galois modules $F(\mathfrak{m}_L)$ in the special cases of wild extension L/K.

The talk is based on results of [1-3]. The present work is supported by the Russian Science Foundation, grant 16-11-10200.

References

[1] S.V. Vostokov and I.I. Nekrasov, *LubinTate formal module in a cyclic unramified pp-extension as Galois module*, J. Math. Sci. (N. Y.), **219** (2016), no. 3, 375-379.

[2] S.V. Vostokov, I.I. Nekrasov, and R.P. Vostokova, *Lutz filtration as a Galois module*, Lobachevskii J. Math. **37** (2016), no. 2, 214–221.

[3] S.V. Vostokov and I.I. Nekrasov, *Cohomology of formal modules over local fields*, to appear.

Chebyshev Lab., St. Petersburg State University
14th Line 29B, Vasilyevsky Island, St. Petersburg 199178, Russia
E-mail: sergei.vostokov@gmail.com, geometr.nekrasov@yandex.ru

Betti tables over short Gorenstein algebras

Roger Wiegand

Let k be a field and $R = R_0 \oplus R_1 \oplus R_2$ be a short Gorenstein graded k-algebra. "Short Gorenstein" means that $\dim_k R_0 = 1 = \dim_k R_2$. We assume that the embedding dimension $e = \dim_k R$ is at least three. The category of finitely generated graded R-modules is known to be wild, but nonetheless we can characterize and classify Betti tables over R. In this talk I will describe the monoid of Betti tables and demonstrate its spectacular lack of factoriality. This is joint work with Luchezar Avramov and Courtney Gibbons.

Department of Mathematics, University of Nebraska at Lincoln
203 Avery Hall PO BOX 880130, Lincoln, NE 68588-0130, USA
E-mail: rwiegand1@unl.edu

Prime ideals in rings of power series and polynomials

Sylvia Wiegand

We discuss prime spectra of low-dimensional commutative Noetherian rings as partially ordered sets. We focus on rings of power series and polynomials. This is primarily joint work with Ela Celikbas and Christine Eubanks-Turner.

Department of Mathematics, University of Nebraska at Lincoln
203 Avery Hall PO BOX 880130, Lincoln, NE 68588-0130, USA
E-mail: swiegand1@unl.edu

On the sandpile group of a subdivision of wheels

Raza Zahid

The combinatorial structure of the sandpile group of modified wheels $\widehat{W}_n$ by using a variant of the chip-firing game [1] has been investigated in this paper. The family of modified wheels $\widehat{W}_n$ is defined by taking the simple wheel graph W_n with n rim vertices and then adding an extra vertex on each of the rim vertices.

References

[1] N.L. Biggs, *Chip-firing and the critical group of a graph*, J. Algebr. Comb. **9** (1999), 25–45.

Department of Mathematics, College of Sciences, University of Sharjah
University City, P.O. Box 27272, Sharjah, United Arab Emirates
E-mail: zraza@sharjah.ac.ae